Math–448. 4–th Homework. Due Friday, April 16, 2004.

1. Given a random sample of size n from the density

$$f(y|\theta) = \begin{cases} \frac{\alpha\theta^\alpha}{y^{\alpha+1}} & \text{if } y > \theta, \\ 0 & \text{else.} \end{cases}$$

where $\theta > 0$ and $\alpha > 0$ are two unknown parameters. Find two jointly sufficient statistics of θ and α.

2. Given a random sample of size n from the density

$$f(y|\theta) = \begin{cases} \frac{3y^5 e^{-\frac{y^3}{\theta}}}{\theta^2} & \text{if } 0 < y, \\ 0 & \text{else.} \end{cases}$$

where $\theta > 0$ is unknown parameter.
(a) Find a sufficient statistic of θ.
(b) Find the MVUE of θ (find a function of the sufficient statistic which an unbiased estimator of θ).
(b) Find the MVUE of θ^2 (find a function of the sufficient statistic which an unbiased estimator of θ).

3. Given a random sample of size n from the density

$$f(y|\theta) = \begin{cases} \frac{\alpha y^{\alpha-1}}{\theta^\alpha} & \text{if } 0 < y < \theta, \\ 0 & \text{else.} \end{cases}$$

where $\theta > 0$ is unknown parameter and $\alpha > 0$ is known.
(a) Find a sufficient statistic of θ.
(b) Find the MVUE of θ.
(b) Find the MVUE of $\mathrm{Var}_\theta(Y)$.

4. Given a random sample of size n from the density

$$f(y|\theta) = \begin{cases} e^{-(y-\theta)}, & \text{if } y \geq \theta, \\ 0 & \text{else.} \end{cases}$$

where $\theta \in \mathbb{R}$ is an unknown parameter.
(a) Find a sufficient statistic of θ.
(b) Find the MVUE of θ.
(b) Find the MVUE of θ^2.

5. Let $Y_1, \ldots, Y_n$ denote a random sample from the density

$$f(y|\theta) = \begin{cases} \theta y^{\theta-1} & \text{if } 0 < y < 1, \\ 0 & \text{else.} \end{cases}$$

where $\theta > 0$.
(a) Find the method of moments estimator of θ. Prove that this estimator consistent of θ.
(b) Find the maximum likelihood estimator of θ. Prove that this estimator consistent of θ.

6. Let $Y_1, \dots, Y_n$ denote a random sample from the density

$$f(y|\theta) = \begin{cases} \frac{2y^3 e^{-\frac{y^2}{\theta^2}}}{\theta^4} & \text{if } 0 < y, \\ 0 & \text{else.} \end{cases}$$

where $\theta > 0$.
(a) Find the method of moments estimator of θ. Is this estimator consistent? (prove it!)
(b) Find the maximum likelihood estimator of θ. Is this estimator consistent? (prove it!)

7. Given a random sample of size n from the density

$$f(y|\theta) = \begin{cases} \frac{\alpha y^{\alpha-1}}{\theta^\alpha} & \text{if } 0 < y < \theta, \\ 0 & \text{else.} \end{cases}$$

where $\theta > 0$ is unknown parameter and $\alpha > 0$ is known.
(a) Find the method of moments estimator of θ. Is this estimator consistent? Is this estimator unbiased for θ? (prove it!)
(b) Find the maximum likelihood estimator of θ. Is this estimator consistent? Is this estimator unbiased for θ? (prove it!)
(c) Which estimator should be preferred?

8. Given a random sample of size n from the density

$$f(y|\theta) = \begin{cases} e^{-(y-\theta)}, & \text{if } y \geq \theta, \\ 0 & \text{else.} \end{cases}$$

where $\theta \in I\!R$ is an unknown parameter.
(a) Find the method of moments estimator of θ. Is this estimator consistent? Is this estimator unbiased for θ? (prove it!)
(b) Find the maximum likelihood estimator of θ. Is this estimator consistent? Is this estimator unbiased for θ? (prove it!)
(c) Which estimator should be preferred?

9. Given a random sample of size n from the density

$$f(y|\theta) = \begin{cases} \frac{m y^{m-1} e^{-\frac{y^m}{\theta}}}{\theta} & \text{if } 0 < y, \\ 0 & \text{else.} \end{cases}$$

where $\theta > 0$ is unknown parameter and $m > 0$ is known. Show that the maximum likelihood estimator of θ is $\hat{\theta} = \frac{\sum_{i=1}^n Y_i^m}{n}$. Show that an approximate large sample $(1-\alpha)\%$ confidence interval for θ is $\hat{\theta} \pm z_{\alpha/2}\frac{\hat{\theta}}{\sqrt{n}}$.

10. Let $Y_1, \dots, Y_m$ denote a random sample of size n from a Poisson distribution with mean λ. Find an asymptotic $100(1-\alpha)$ % confidence interval of $t(\lambda) = e^{-\lambda} = P(Y = 0)$.