Homework 5

1. Let p = proportion of dentist who recommend a certain brand of gum

$H_0: p = 0.75 \quad H_a: p < 0.75$

$\hat{p} = \frac{y}{n} = \frac{273}{390} = 0.7$

$$\frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{0.7 - 0.75}{\sqrt{\frac{0.75(1-0.75)}{390}}} = -2.28$$

The p-value is $P(N(0,1) \le -2.28) = 0.0113$

Reject H_0 for $\alpha = 0.10$ and 0.05

Accept H_0 if $\alpha = 0.01$

If $\alpha = 0.05$, we reject H_0 if $\hat{p} \le p_0 - z_\alpha \sqrt{\frac{p_0(1-p_0)}{n}}$

$$= 0.75 - 1.645\sqrt{\frac{(0.75)(1-0.75)}{390}} = 0.713931$$

The type II errors are

$p = 0.70 \quad P_{0.70}(\hat{p} > 0.713931) = P\left(N(0,1) \geq \frac{0.713931 - 0.70}{\sqrt{\frac{0.70(1-0.70)}{390}}}\right)$

$= P(N(0,1) \geq 0.6) = 0.2743$

$p = 0.65 \quad P_{0.65}(\hat{p} > 0.713931) = P\left(N(0,1) \geq \frac{0.713931 - 0.65}{\sqrt{\frac{0.65(1-0.65)}{390}}}\right)$

$= P(N(0,1) \geq 2.65) = 0.004$

$p = 0.60 \quad P_{0.6}(\hat{p} > 0.71391) = P(N(0,1) \geq 4.59) = 0.00$

2. p_1 = proportion of households with pet dogs who were burglarized

p_2 = without

$H_0: p_1 = p_2$ $\quad H_a: p_1 < p_2$

Reject H_0 if $\frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} \le K$

$\hat{p}_1 = \frac{24}{250} = 0.096 \qquad \hat{p}_2 = 0.1355$

$\hat{p} = \frac{24+29}{250+214} = 0.114224$

$$\frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} = \frac{0.096 - 0.1355}{\sqrt{0.1142(1-0.1142)\left(\frac{1}{250}+\frac{1}{214}\right)}} = -1.33$$

p-value $= P(N(0,1) \le -1.33) = 0.0918$

Reject H_0, if $\alpha = 0.10$

Accept H_0, if $\alpha = 0.05$ and $\alpha = 0.01$

3. $H_0: \mu = 110 \quad H_a: \mu > 110, \quad \sigma^2 = 100$

$$\frac{\bar{Y} - \mu_0}{\sigma/\sqrt{n}} = \frac{113.5 - 110}{10/\sqrt{16}} = 1.4$$

The p-value is $P(N(0,1) \geq 1.4) = 0.0808$

Reject H_0, if $\alpha = 0.10$

Accept H_0, if $\alpha = 0.05$ and $\alpha = 0.01$

We reject H_0, if $\bar{Y} \geq \mu_0 + z_\alpha \frac{\sigma}{\sqrt{n}} = 110 + (1.645)\frac{10}{\sqrt{16}} = 114.1125$

type II error is $P_{\mu_a}(\bar{Y} \leq 114.1125) = P\left(N(0,1) \leq \frac{114.1125 - \mu_0}{10/\sqrt{16}}\right)$

If $\mu_a = 115$

type II error is $P(N(0,1) \leq -0.355) = 0.36$

If $\mu_a = 120$

type II error is $P(N(0,1) \leq -2.355) = 0.0094$

If $\mu_a = 125$

type II error is $P(N(0,1) \leq -4.35) = 0.00$

4. μ_1 = average starting salary of industrial engineering graduates

μ_2 = average starting salary of civil engineering graduates

$n_1 = 16 \quad \bar{y}_1 = 47700 \quad s_1 = 2400$

$n_2 = 16 \quad \bar{y}_1 = 46400 \quad s_2 = 2200$

$H_0: \mu_1 = \mu_2 \qquad H_a: \mu_1 > \mu_2$

$$\text{p-value} = P\left(\frac{\bar{y}_1 - \bar{y}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \geq \frac{47700 - 46400}{\sqrt{\frac{(2400)^2}{16} + \frac{(2200)^2}{16}}}\right)$$

$= P(N(0,1) \geq 1.597) = 0.0548$

We reject H_0, if $\alpha = 0.10$

Accept H_0 if $\alpha = 0.05$ and $\alpha = 0.01$

5. $\sum Y_i = 24+28+21+23+32+22 = 150$

$\bar{y} = \frac{150}{6} = 25$

$\sum Y_i^2 = 24^2 + 28^2 + 21^2 + 23^2 + 32^2 + 22^2 = 3838$

$\sum (Y_i - \bar{Y})^2 = \sum Y_i^2 - n(\bar{Y})^2 = 3838 - 6(25)^2 = 88$

$s^2 = \frac{88}{5} = 17.6$

To test $H_0: \mu = 29$ versus $H_a: \mu < 29$

We reject H_0 if $\frac{\bar{Y} - \mu_0}{S/\sqrt{n}} \le -t_\alpha$

$$\frac{\bar{Y} - \mu_0}{S/\sqrt{n}} \le -t_\alpha$$

$$\frac{(25-29)\sqrt{6}}{\sqrt{17.6}} = -2.33$$

$t_{0.05} = 2.015$ $\qquad t_{0.025} = 2.517$

So the p-value satisfies

$$0.05 > \text{p-value} > 0.025$$

the rejection region

is $\{(Y_1 - Y_6): \frac{\sqrt{6}(\bar{Y} - 29)}{S} \le -3.365\}$

6. $\sum Y_i = 16698$ $\bar{Y} = \frac{\sum Y_i}{n} = \frac{16698}{25} = 667.92$

$\sum Y_i^2 = 11724788$

$\sum (Y_i - \bar{Y})^2 = \sum Y_i^2 - n(\bar{Y})^2 = 11724788 - (25)(667.92)^2 = 571859.84$

$s^2 = \frac{\sum (Y_i - \bar{Y})^2}{n-1} = \frac{571859.84}{24} = 23827$

Reject H_0 if $\frac{(n-1)s^2}{\sigma_0^2} \geq \chi^2_\alpha = 36.41$ $s^2 \geq 2484.83$

$\frac{(n-1)s^2}{\sigma_0^2} = \frac{24(23827)}{140^2} = 29.14$

$\chi^2_{0.1} = 33.1963$ $\chi^2_{0.9} = 15.65$

$0.9 > \text{p-value} > 0.10$

7. $H_0: \sigma_1^2 = \sigma_2^2$ versus $H_a: \sigma_1^2 \neq \sigma_2^2$ $n_1 = 16$ $n_2 = 12$

We reject H_0 if $\frac{s_1^2}{s_2^2} \geq F_{n_1-1, n_2-1}(\frac{\alpha}{2})$ or $\frac{s_1^2}{s_2^2} \leq F_{n_1-1, n_2-1}(\frac{\alpha}{2})$

$= \frac{1}{F_{n_2-1, n_1-1}(\frac{\alpha}{2})}$ $F_{n_2-1, n_1-1}(\frac{\alpha}{2}) \leq \frac{s_2^2}{s_1^2}$

$\frac{s_1^2}{s_2^2} = \frac{1356.75}{629.21} = 2.15$ $\frac{s_2^2}{s_1^2} = 0.46$

$\underline{\alpha = 0.05}$

Reject H_0 if $\frac{s_1^2}{s_2^2} \geq 3.18$ or $\frac{s_1^2}{s_2^2} \leq \frac{1}{2.86}$

Accept H_0, at the level 0.05.

$\alpha = 0.10$

Reject H_0 if $\frac{s_1^2}{s_2^2} \geq 2.62$ or $\frac{s_1^2}{s_2^2} \leq \frac{1}{2.40}$, Accept H_0

$\alpha = 0.20$

Reject H_0 if $\frac{s_1^2}{s_2^2} \geq 2.10$ or $\frac{s_1^2}{s_2} \leq \frac{1}{1.97}$, Reject H_0

$0.20 \geq p\text{-value} \geq 0.10$

8. $H_0: \mu = 30000$ $\quad H_a: \mu = 35000$

$$0.01 = P_{\mu=30{,}000}(\bar{Y} \geq K) = P\left(N(0,1) \geq \frac{K-30000}{5000/\sqrt{n}}\right) = P(N(0,1) \geq 2.326$$

$$0.02 = P_{\mu=35000}(\bar{Y} \leq K) = P\left(N(0,1) \leq \frac{\sqrt{n}(K-35000)}{5000}\right) = P(N(0,1) \geq 2.054)$$

$$\frac{\sqrt{n}(K-30000)}{5000} = 2.326$$

$$\frac{\sqrt{n}(35000-K)}{5000} = 2.054$$

$$n = \frac{(5000)^2(2.325+2.054)^2}{(35000-30000)^2} = 19.18 \qquad n=20$$

$$K = 3000 + \frac{(2.326)(5000)}{\sqrt{20}} = 32600.55$$

9.

p_1= proportion of criminals in normal males.

p_2= proportion of criminals in males with abnormal chromosome

$H_0: p_1 = p_1 \quad H_a: p_1 < p_2$

$\hat{p}_1 = \frac{381}{4098} = 0.0930 \qquad \hat{p}_2 = \frac{8}{28} = 0.2857$

$\hat{p} = \frac{381+8}{4098+28} = 0.094280$

p-value = $P(\hat{p}_1 - \hat{p}_2 \leq 0.0930 - 0.28571$

$= P\left(N(0,1) \leq \dfrac{0.0930 - 0.2857}{\sqrt{(0.094280)(1-0.094280)\left(\frac{1}{4098}+\frac{1}{28}\right)}}\right.$

$= P(N(0,1) \leq -3.477) = 0.000$

Reject H_0, if $\alpha = 0.10$, $\alpha = 0.05$ and $\alpha = 0.01$

10. $H_0: \mu = 18.5$ $\quad H_a: \mu = 20.$

$$0.05 = P_{18.5}(\bar{Y} \geq k) = P\left(N(0,1) \geq \frac{\sqrt{n}(k-18.5)}{2.4}\right)$$

$$0.95 = P_{20}(\bar{Y} \geq k) = P\left(N(0,1) \geq \frac{\sqrt{n}(k-20)}{2.4}\right)$$

$$\left.\begin{array}{l} 1.645 = \sqrt{n}\dfrac{(k-18.5)}{2.4} \\ \\ -1.645 = \dfrac{\sqrt{n}(k-20)}{2.4} \end{array}\right\} \quad \frac{\sqrt{n}(20-18.5)}{2.4} = 2(1.645)$$

$$n = \frac{(2(1.645))^2(2.4)^2}{(20-18.5)^2} = 27.70.$$