Math–448. 5–th Homework. Due Monday, May 3, 2004.

1. It is claimed than 75 % of all dentists recommend a certain brand of gum for their gum chewing patients. A consumer group doubted this claim believing that the proportion is lower and decided made a sample survey. A survey of 390 dentists found that 273 recommended this brand of gum.

   (a) What are the null and alternative hypothesis?

   (b) Find the p value of the test.

   (c) Which hypothesis would you accept if the significance level is (a) $\alpha = 0.10$? (b) $\alpha = 0.05$? (c) $\alpha = 0.01$?

   (d) For the test with $\alpha = 0.05$. What is the type II error if (a) $p = 0.70$? (b) $p = 0.65$? (b) $p = 0.60$?

2. A report on the nightly news broadcast stated that 24 out of 250 households with pet dogs were burglarized and 29 out of 214 without pet dogs were burglarized. Some experts believe that dog ownerships is associated with decreased in the number of breakings .

   (a) Do these data lend support to this belief? What are the null and alternative hypothesis?

   (b) Find the appropriate p-value.

   (c) Which hypothesis would you accept if the significance level is (a) $\alpha = 0.10$? (b) $\alpha = 0.05$? (c) $\alpha = 0.01$?

3. Assume that IQ scores for a certain population are approximately normally distributed with mean μ and variance 100. To test $H_0 : \mu = 110$ against the one–sided alternative hypothesis $H_a : \mu > 110$, we take a random sample of size $n = 16$ from this population and observe $\bar{y} = 113.5$.

   (a) Find the p value of the test.

   (b) Which hypothesis would you accept if the significance level is (a) $\alpha = 0.10$? (b) $\alpha = 0.05$? (c) $\alpha = 0.01$?

   (c) Find the rejection region for the test at the significance level $\alpha = 0.05$. What is the type II error if (a) $\mu = 115$? (b) $\mu = 120$? (c) $\mu = 125$?

4. A professor claims that the average starting salary of industrial engineering graduating seniors is greater than that of civil engineering graduates. To study this claim, samples of 16 industrial engineers and 16 civil engineers, all of whom graduate in 1993, were chosen and sample members were queried about their starting salaries.

   (a) If the industrial engineers had a sample mean salary of \$47,700 and sample standard deviation of \$2,400 and the civil engineers had a sample mean salary of \$46,400 and sample standard deviation of \$2,200 has the professor's claim been verified? What are the null and alternative hypothesis?

(b) Find the appropriate p-value.

(c) Which hypothesis would you accept if the significance level is (a) $\alpha = 0.10$? (b) $\alpha = 0.05$? (c) $\alpha = 0.01$?

5. A car company claims that its new experimental engine runs 29 minutes with one gallon of fuel. Test runs with an experimental engine it operated, respectively, for 24, 28, 21,23, 32, and 22 minutes with one gallon of fuel. Assume that the data is a random sample from a normal distribution. Is there enough evidence at the .01 level to claim that the new experimental engine runs less than 29 minutes?

   (a) What are the null and alternative hypothesis?

   (b) Determine the rejection region.

   (c) Estimate the p value of the test.

6. Let Y equal the number of pounds of butterfat produced by a Holstein cow during the 305–day milking period following the birth of a calf. We assume that the distribution of Y is normal with mean μ and variance σ^2. The following data was obtained:

425	710	661	664	732	714	934	761	744
653	725	657	421	573	535	602	537	
405	874	791	721	849	567	468	975	

   Assume that the data is a random sample from a normal distribution.

   (a) Test the null hypothesis $H_0 : \sigma^2 = 140^2$ against the alternative $H_a : \sigma^2 > 140^2$ at the level $\alpha = 0.05$. Determine the rejection region.

   (b) Do we accept or reject the null hypothesis at the level $\alpha = 0.05$.

   (c) Find the p–value of the data.

7. Let Y_1 and let Y_2 denote the weights in grams of male and female gallinules, respectively. Assume that both distributions are normally distributed. Given that $n_1 = 16$, $\bar{Y}_1 = 415.16$, $s_1^2 = 1356.75$, $n_2 = 13$, $\bar{Y}_2 = 347.4$, $s_2^2 = 629.21$. We test the null hypothesis $H_0 : \sigma_1^2 = \sigma_2^2$ against the alternative $H_a : \sigma_1^2 \neq \sigma_2^2$ at the level $\alpha = 0.05$.

   (a) Determine the rejection region.

   (b) Do we accept or reject the null hypothesis a the level $\alpha = 0.05$?

   (c) Estimate the p–value for this test.

8. Let us assume that the life of a tire in miles, say Y, is normally with distributed with mean μ and standard deviation 5000. Past experience indicates that $\mu = 30,000$. The manufacturer claims that the tires made by a new process have mean $\mu > 30,000$, and it is very possible that $\mu = 35,000$. We shall observe n independent values of Y, say $y_1, \ldots, y_n$, and we shall reject H_0 if $\bar{y} \geq k$. Determine n and k so that the type I error is 0.01 and the type II error when $\mu = 35,000$ is 0.02.

9. A study of chromosome abnormalities and criminality examined date on 4124 Danish males born in Copenhagen. Each man was classified as having a criminal record or not, using the penal registers maintained in the offices of the local police chiefs. Each was also classified as having the normal male XY chromosome pair or one of the abnormalities XYY or XXY. Of the 4096 men with normal chromosomes, 381 had criminal records, while 8 of the 28 men with chromosome abnormalities had criminal records. Some experts believe that chromosome abnormalities are associated with increased I criminality. Do these data lend support to this belief?

   (a) What are the null and alternative hypothesis?

   (b) Find the appropriate p-value.

   (c) Which hypothesis would you accept if the significance level is (a) $\alpha = 0.10$? (b) $\alpha = 0.05$? (c) $\alpha = 0.01$?

10. The assembly time in a plant is a normal random variable with mean 18.5 seconds and standard deviation 2.4 seconds. Consider the test $H_0 : \mu = 18.5$ versus $H_a : \mu > 18.5$ at the significance level 0.05. It is very important that the assembly time not exceed 20 seconds. How large a sample is necessary to reject $H_0 : \mu = 18.5$ with probability 0.95 if $\mu = 20$.