Math–448. Practice Problems for the 2nd Exam.

1. Let $Y_1, \ldots, Y_{10}$ be a random sample from a N(0,1) distribution. Find the distribution of

$$\frac{\sum_{i=1}^{5} Y_i}{\sqrt{\sum_{j=6}^{10} Y_j^2}}.$$

2. Given a random sample of size n from the density

$$f(y) = \begin{cases} \frac{4\theta^4}{y^3} & \text{if } \theta < y, \\ 0 & \text{else.} \end{cases}$$

where $\theta > 0$ is unknown parameter. Find constants a and b such that $\hat{\theta}_1 = a\bar{Y}$ and $\hat{\theta}_2 = bY_{(1)}$ are both unbiased estimators of θ. Find the mean square error of $\hat{\theta}_1$ and $\hat{\theta}_2$? Find the efficiency of $\hat{\theta}_1$ relative to $\hat{\theta}_2$. Show that $\hat{\theta}_1$ relative to $\hat{\theta}_2$ are both consistent estimators of θ. Which of the estimators $\hat{\theta}_1$ and $\hat{\theta}_2$ has smallest mean square error? Which estimator is preferred?

3. Given a random sample of size n from a distribution with mean μ and variance σ^2. Show that $\frac{1}{n}\sum_{j=1}^{n}(Y_j - \bar{Y})^2$ and $\frac{1}{n-1}\sum_{j=1}^{n}(Y_j - \bar{Y})^2$ are both consistent estimators of σ^2.

4. Given a random sample of size n from the density

$$f(y|\theta) = \begin{cases} \frac{\theta}{y^2} & \text{if } \theta < y, \\ 0 & \text{else,} \end{cases}$$

where $\theta > 0$ is unknown parameter. Find a constant a such that $\hat{\theta} = aY_{(1)}$ is an unbiased estimator of θ. Find the MSE of $\hat{\theta}$. Show that $\hat{\theta}$ is a consistent estimator of θ.

5. Given a random sample of size n from the density

$$f(y|\theta) = \begin{cases} \frac{1}{\theta}e^{-\frac{y}{\theta}} & \text{if } y > 0, \\ 0 & \text{else,} \end{cases}$$

where $\theta > 0$ is unknown parameter. Find a constant a such that $\hat{\theta} = a\frac{1}{n}\sum_{j=1}^{n}Y_j^2$ is an unbiased estimator of θ^2. Find the MSE of $\hat{\theta}$. Show that $\hat{\theta}$ is a consistent estimator of θ^2.

6. A gas station wants to know an interval estimate at the 90 % confidence level for the mean price μ for regular gasoline in the Greater Binghamton area. A sample of size 24 has an average price of $1.79. If σ is 6 cents, compute the confidence interval.

7. In April of 2002 a CNN poll stated that 70% of a random sample of 1009 Americans surveyed by telephone were concerned that suicide bombers might strike the U.S. Find a confidence interval at the 99 % level.

8. In a recent national poll, 54% of 900 adults surveyed stated that they favored President Bush over John Kerry in the next presidential election. Find a 95% confidence interval for the true proportion of adults nationwide who favored Bush at that time.

9. In a college of 2650 students, 264 out of 400 surveyed had registered to vote. Find a 90% confidence interval for the true proportion of those registered to vote at this college.

10. On June 25, 1995, The Associated Press reported the findings of a national survey conducted by the Center for Social and Religious Research at the Hartford Seminary. The study was on the divorce rate of a group of 5000 Protestant clergymen and 5000 Protestant clergywomen. It was found that 20% out of 2086 clergymen responding had been divorced at least once. Find a 99% confidence interval for the true proportion of those that have been divorced among the targeted group of 5000 clergymen.

11. State University New York claims the average starting salary of its graduates is $38,500. A sample of 100 SUNY students is sampled and yields an average starting salary of $36,800 with a standard deviation of $9,369. Using a 95% confidence level what can you say about SUNY's claim?

12. If of 200 people, randomly selected from those with unusual names, 60 believe their name might have been a factor in career choice, give the 90% confidence interval for the population proportion who believe their name might have been a factor in career choice.

13. How many test runs of an automobile are required for determining its average miles-per-gallon (mpg) rating on the highway to within ± 0.5 mpg with 95% confidence? The standard deviation of the population of miles per gallon is $\sigma = 5$.

14. A business school researcher is interested in the mean amount of time that hotel managers have stayed with their current employers. She plans to take a random sample of hotel managers, and ask each how long he or she has stayed with the current employer. A reasonable guess for the population variance of staying time for all hotel managers is 4. If a margin of error of 1.5 and a confidence level of 86 percent are required, how large a random sample will be needed?

15. Suppose that you were interested in the average number of units that students take at a two year college to get an AA degree. Suppose you wanted to find a 95% confidence interval with a margin of error of .5 for μ knowing that $\sigma = 10$. How many people should we ask?

16. Suppose that you are in charge to see if dropping a computer will damage it. You want to find the proportion of computers that break. If you want a 90% confidence interval for this proportion, with a margin of error of $\pm 4\%$, How many computers should you drop?

17. In 1882, Michelson performed an experiment to measure the speed of light. He obtained 23 measurements of the speed of light in km/sec. The mean of his measurements is 299715.1 km/sec. The standard deviation of his measurements is 158.2 km/sec.

18. How many test runs of an automobile are required for determining its average miles-per-gallon (mpg) rating on the highway to within 0.025 mpg with 90% confidence? The standard deviation of the population of miles per gallon is $\sigma = 5$.

19. A random sample of size $n = 25$ is taken from a normally distributed population with unknown mean μ and standard deviation $\sigma = 17$. What is the probability that the mean of the 25 observations is more than 4 away from the mean μ?

20. A cement packing plant fills bags with cement. The weight, Y, of a randomly chosen bag is normally distributed with mean 50kg and variance 4kg^2. A random sample of 9 bags is taken. Find the probability that the sample mean is between 47.6 and 52.4.

21. District court records provided data on sentencing for 19 criminal convicted of negligent homicide. The mean and standard deviation of the sentences were found to be 72.7 and 10.2 months, respectively. Determine a 95% confidence interval for the mean sentence for this crime.

22. A random sample of $n = 10$ packs off of the factory line yields $s^2 = 4.2$ g. Determine a 95% confidence interval for the variance of the packs .

23. What is the difference between commuting patterns for students and professors? 11 students and 14 professors took part in a study to find mean commuting distances. The mean number of miles traveled by students was 5.6 and the standard deviation was 2.8. The mean number of miles traveled by professors was 14.3 and the standard deviation was 9.1. Construct a 95% confidence interval for the difference between the means. Assume that the variances of the mean number of miles traveled by students and professors are equal.

24. 300 men and 400 women we asked how they felt about taxing Internet sales. 75 of the men and 90 of the women agreed with having a tax. Find a 90% confidence interval for the difference between the proportion of men favoring the tax and the proportion of women favoring the tax.

25. An economist wishes to estimate the average family income in a certain population. The population standard deviation is known to be $4,500, and the economist uses a random sample of size $n = 225$. What is the probability that the sample mean will fall within $600 of the population mean?

26. Suppose 25 people take an IQ test and that their scores are normally distributed. If $s^2 = 100$, find a 95% upper-CI for the variance σ^2.

27. Suppose 25 people take an IQ test A and 16 people take IQ test B. Assume all scores are normal and independent. If $s_1^2 = 100$ and S2B $s_2^2 = 70$, find a 95% upper-CI for the ratio of variances $\frac{\sigma_1^2}{\sigma_2^2}$.

28. The probability that a student correctly answers a certain test question is p. Suppose a random sample of 100 students yields 40 correct answers to the question. Find a 90% 2-sided CI for p.