1. $\frac{1}{\sqrt{5}}\sum_{i=1}^{5} Y_i \sim N(0,1)$, $\sum_{j=6}^{10} Y_j^2 \sim \chi^2(5)$

So, $$\frac{\frac{1}{\sqrt{5}}\sum_{i=1}^{5} Y_i}{\sqrt{\frac{1}{5}\sum_{j=6}^{10} Y_j^2}} = \frac{\sum_{i=1}^{5} Y_i}{\sqrt{\sum_{i=1}^{5} Y_i^2}} \sim t(5)$$

2. $$f(y) = \begin{cases} \frac{4\theta^4}{y^5} & \text{if } \theta < y \\ 0 & \text{else} \end{cases}$$

$$E[Y] = \int_\theta^\infty y \frac{4\theta^4}{y^5}\,dy = \int_\theta^\infty 4\theta^4 y^{-4}\,dy = \frac{-4}{3}\theta^4 y^{-3}\Big]_\theta^\infty = \frac{4}{3}\theta$$

$$E[Y^2] = \int_\theta^\infty y^2 \frac{4\theta^4}{y^5}\,dy = \int_\theta^\infty 4\theta^4 y^{-3}\,dy = \frac{-4\theta^4 y^{-2}}{2}\Big]_\theta^\infty = 2\theta^2$$

$$Var(Y) = 2\theta^2 - \left(\frac{4}{3}\theta\right)^2 = \left(2 - \frac{16}{9}\right)\theta^2 = \frac{2\theta^2}{9}$$

$a = \frac{3}{4}$ $\quad \frac{3}{4}\bar{Y}$ is an unbiased estimator of θ

$$MSE\left(\frac{3\bar{Y}}{4}\right) = \frac{9}{16}\frac{2\theta^2}{9} = \frac{\theta^2}{8}$$

$$F(y) = 1 - \frac{\theta^4}{y^4}$$

$$f_{Y_{(1)}}(y) = n\left(1-F(y)\right)^{n-1} f(y) = n\left(\frac{\theta^4}{y^4}\right)^{n-1}\frac{4\theta^4}{y^5} = \frac{4n\,\theta^{4n}}{y^{4n+1}}, \text{ if } \theta < y$$

$$E[Y_{(n)}] = \int_\theta^\infty y \, \frac{4n\,\theta^{4n}}{y^{4n+1}}\,dy = \int_\theta^\infty 4n\,\theta^{4n}\, y^{-4n}\,dy$$

$$= 4n\,\frac{\theta^{4n}\, y^{-4n+1}}{-4n+1}\Big]_\theta^\infty = \frac{4n\,\theta}{4n+1}$$

$$E[Y_{(n)}^2] = \int_\theta^\infty y^2\,\frac{4n\,\theta^{4n}}{y^{4n+1}}\,dy = \int_\theta^\infty 4n\,\theta^{4n}\, y^{-4n+1}\,dy$$

$$= 4n\,\theta^{4n}\,\frac{y^{-4n+2}}{-4n+2}\Big|_\theta^\infty = \frac{4n\,\theta^2}{4n-2}$$

$$Var(Y_{(n)}) = \frac{4n\theta}{4n-2} - \left(\frac{4n\theta}{4n-1}\right)^2 = \frac{\theta\, 4n}{(4n-2)(4n-1)^2}\left((4n-1)^2 - 4n(4n-2)\right)$$

$$= \frac{4n\theta}{(4n-2)(4n-1)^2}$$

$b = \frac{4n-1}{4n}$ $\qquad \frac{4n-1}{4n} Y_{(n)}$ is an unbiased estimator of θ

$$MSE\left(\frac{4n-1}{4n} Y_{(1)}\right) = \frac{\theta^2}{4n(4n-2)}$$

$$eff(\hat\theta_1, \hat\theta_2) = \frac{\frac{\theta^2}{4n(4n-2)}}{\frac{\theta^2}{8}} = \frac{2}{n(4n-2)}$$

$\bar{Y} \xrightarrow{P} \frac{4}{3}\theta$ $\qquad \frac{3\bar{Y}}{4} \xrightarrow{P} \theta$, $\frac{3\bar{Y}}{4}$ is a consistent estimator of θ

Given $\varepsilon > 0$, $0 < \varepsilon < \theta$

$$P\left(\left| \frac{4n-1}{4n} Y_{(1)} - \theta \right| \geq \varepsilon \right) = P\left(\frac{4n-1}{4n} Y_{(1)} - \theta > \varepsilon \right)$$

$$+ P\left(\frac{4n-1}{4n} Y_{(1)} - \theta < -\varepsilon \right)$$

$$= P\left(Y_{(1)} > \frac{4n}{n-1} (\theta + \varepsilon) \right) + P\left(Y_{(1)} < \frac{4n}{4n-1} (\theta - \varepsilon) \right)$$

$$\theta - \varepsilon < \frac{4n-1}{4n} \theta$$

$$\frac{4n}{4n-1} (\theta - \varepsilon) < \theta$$

$$= \int_{\frac{4n}{4n-1}(\theta+\varepsilon)}^{\infty} \frac{4n\, \theta^{4n}}{y^{4n+1}} \, dy = \frac{\theta^{4n}}{\left(\frac{4n}{4n-1} (\theta + \varepsilon) \right)^{4n}} = \left(\frac{4n-1}{4n} \right)^{4n} \left(\frac{\theta}{\theta + \varepsilon} \right)^{4n} \longrightarrow 0$$

6. $\bar{y} = 1.74 \quad n = 24, \quad \sigma = 6$

$$\bar{y} \pm z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} = 1.76 \pm (1.645)\frac{(6)}{\sqrt{24}} = 1.76 \pm 2.0147$$

$$= (-0.2547,\ 3.7747)$$

7. $$\hat{p} \pm z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 0.7 \pm (2.575)\sqrt{\frac{(0.7)(0.3)}{1009}} = 0.7 \pm 0.0372$$

$$= (0.6628,\ 0.7372)$$

8. $$\hat{p} \pm z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 0.54 \pm (1.96)\sqrt{\frac{(0.54)(1-0.54)}{900}} = 0.54 \pm 0.0326$$

$$= (0.5074,\ 0.5726)$$

11. $\bar{y} = 36800 \quad S = 9364 \qquad \bar{y} \pm z_{\frac{\alpha}{2}} \frac{S}{\sqrt{n}}$

$$36800 \pm (1.96)\frac{9364}{\sqrt{100}} = 36800 \pm 1836.32$$

$$= (34963.7,\ 38636.3)$$

13. $$0.5 = z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} = (1.96)\frac{(5)}{\sqrt{n}}$$

$$n = \left(\frac{(1.96)(5)}{0.5}\right)^2 = (19.6)^2 = 384.16$$

16. $$0.04 = z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \frac{1.69}{2}\frac{1}{\sqrt{n}}$$

$$n = \left(\frac{1.69}{2(0.04)}\right)^2 = 446.26$$

19. $n=25$

$P(-4 \le \bar{Y}-\mu \le 4) = P\left(\frac{-4\sqrt{25}}{17} \le N(0,1) \le \frac{4\sqrt{25}}{17}\right)$

$= P(-1.17 \le N(0,1) \le 1.17) = 1-2(0.1210) = 0.758$

20. $\mu = 50$ $\sigma^2 = 4$

$P(47.6 \le \bar{y} \le 52.4) = P\left(\frac{(47.6-50)\sqrt{9}}{\sqrt{4}} \le N(0,1) \le \frac{\sqrt{9}(52.4-50)}{\sqrt{4}}\right)$

$= P(-3.6 \le N(0,1) \le 3.6) = 1$

21. $n=19$ $\bar{y} = 72.7$ $s = 10.2$

$\bar{y} \pm t_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}} = 72.7 \pm (2.101)\frac{(10.2)}{\sqrt{19}} = 72.7 \pm 4.9164$

$= (67.784, 77.616)$

22. $n=9$ $\left(\frac{s^2}{\chi^2_{\frac{\alpha}{2}}}, \frac{s^2}{\chi^2_{1-\frac{\alpha}{2}}}\right) = \left(\frac{4.2}{19.0228}, \frac{4.2}{2.70039}\right)$

$= (0.2208, 1.5553)$

23. $n_1 = 11,\ \bar{Y}_1 = 5.6\quad S_1 = 2.8$ $\qquad \alpha = 0.05$

$n_2 = 14,\ \bar{Y}_2 = 14.3\quad S_2 = 9.1$ $\qquad df = 23$

$$\bar{y}_1 - \bar{y}_2 \pm t_{\frac{\alpha}{2}}\, s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}} = 5.6 - 14.5 \pm (2.069)\sqrt{50.2143}\sqrt{\frac{1}{11} + \frac{1}{14}}$$

$$s_p^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1 - 1 + n_2 - 1} = \frac{(10)(2.8)^2 + (13)(9.1)^2}{10 + 13} = 50.2143$$

$$= -8.9 \pm 5.9072 = (-14.8072, -2.9928)$$

24. $$\hat{p}_1 - \hat{p}_2 \pm z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$ $\qquad n_1 = 300 \quad n_2 = 400$

$\hat{p}_1 = \frac{75}{300} = 0.25 \qquad \hat{p}_2 = 0.225 \qquad \alpha = 10\%$

$$0.25 - 0.225 \pm 1.69\sqrt{\frac{(0.25)(1-0.25)}{300} + \frac{(0.225)(1-0.225)}{400}}$$

$$= 0.025 \pm 0.0550 = (-0.03, 0.08)$$