Math–501. First Midterm. Wednesday, September 26, 2007.

1. Write the definition of a σ–field of a sample space Ω.

2. Write the definition of probability measure P defined on a measurable space $(\Omega, \mathcal{F})$.

3. Let $(\Omega, \mathcal{F}, P)$ be a probability space. Let $A_1, \dots, A_n \in \mathcal{F}$, where $n \geq 2$. Write the definition of the independence of $A_1, \dots, A_n$.

4. Suppose that A, B and C are independent events. Suppose that $P[A] = \frac{1}{2}$, $P[B] = \frac{1}{3}$, $P[C] = \frac{1}{4}$. Find $P[(A \cap B) \cup C]$.

5. Let $X_1, \dots, X_m$ be random variables. Let F_j be the cumulative distribution of X_j, for each $1 \leq j \leq m$. Let $\lambda_j \geq 0$, for each $1 \leq j \leq m$ such that $\sum_{j=1}^m \lambda_j = 1$. Show that there exists a r.v. X with cumulative distribution function $G(x) = \sum_{j=1}^m \lambda_j F_j(x)$, $x \in \mathbb{R}$.

6. Let X be a random variable. Suppose that for each $A \in \mathcal{B}(\mathbb{R})$, either $P(A) = 1$, or $P(A) = 0$. Show that there exists a constant c such that $\mathbb{P}(X = c) = 1$.

7. Let X be a random variable. Show that X and $X + 1$ do not have the same distribution.

8. Let X be a random variable. Suppose X and $X + 1$ are independent random variables. Show that there exists a constant c such that $\mathbb{P}(X = c) = 1$.