Math 501. 5–th Homework. Due Friday, November 16, 2007.

Homework on "Conditional Expectation".

1. Suppose that a test for diagnosing gonorrhea is positive with probability 0.95 for a person with gonorrhea while it is positive with probability 0.01 for a person without gonorrhea. The probability that a person has gonorrhea is 0.25. A person is tested for the disease and the test indicates that he/she has it. Find the probability that the person actually has the disease.

2. Let X and Y have the pdf

$$f(x,y) = \begin{cases} 6(y-x) & \text{if } 0 \le x \le y \le 1 \\ 0 & \text{else} \end{cases}$$

   (i) Find $f_{Y|X}(y|x)$.

   (ii) Find $E[Y|X=x]$ and $\mathrm{Var}(Y|X=x)$.

3. An actuary models the lifetime in years of a random selected person as a r.v. X with p.d.f. $f(x) = \frac{6x^5}{90^6}$, for $0 < x < 90$. For a 20–year old, find:
   (i) the probability that he lives above 70 years old.
   (ii) the mean and the standard deviation of his time of death.

4. An insurance company designates its customers as high risk, middle risk and low risk. This insurance company uses the following table:

Type of risk	Proportion of customers	Expected annual claim amounts	Standard deviation of claim amounts
high	50%	2000	1500
middle	30%	1000	500
low	20%	500	100

   Find the mean and the standard deviation of the annual claim amount submitted by a randomly selected customer.

5. Let X and Y be two r.v.'s with joint probability mass function:

$$P[X=0,Y=0] = 0.800, \quad P[X=1,Y=0] = 0.050,$$
$$P[X=0,Y=1] = 0.025, \quad P[X=1,Y=1] = 0.125,$$

   Calculate $E[X|Y=1]$ and $\mathrm{Var}(X|Y=1)$.

6. An actuary estimates that the price X (in thousand of dollars) of a sedan has p.d.f. $f(x) = \frac{1}{25}$, $0 < x < 25$. Given that $X = x$, the total amount (in thousand of dollars) Y of the annual claims submitted by a policyholder has p.d.f. $f_{Y|X}(y|x) = \frac{x+10y}{100(x+500)}$ for $0 \leq y \leq 100$.

   (i) For a \$20000 sedan, what is the probability that total amount Y of the annual claims is worth less than \$10000?

   (ii) What is the probability that total amount Y of the annual claims is worth less than \$10000?

   (iii) For a \$20000 sedan, what is the average total amount of the submitted annual claims?

   (iv) What is the average total amount of the submitted annual claims by a randomly selected policyholder?

7. Suppose that X_1, X_2 and X_3 are independent r.v.'s. Show that $E[X_1X_3|\sigma(X_1, X_2)] = X_1E[X_3]$.

8. Let X be an absolutely continuous r.v. with pdf f_X. Show that

$$E[X|\,|X| = x] = \begin{cases} \frac{f_X(x)-f_X(-x)}{f_X(x)+f_X(-x)}|x|, & \text{if } x > 0 \text{ and } f_X(x) + f_X(-x) > 0, \\ 0 & \text{if } x > 0 \text{ and } f_X(x) + f_X(-x) = 0, \\ 0 & \text{if } x \leq 0, \end{cases}$$