Manual for SOA Exam FM/CAS Exam 2.

Chapter 1. Basic Interest Theory.
Section 1.4. Present value and discount.

Extract from:
"Arcones' Manual for the SOA Exam FM/CAS Exam 2,
Financial Mathematics. Spring 2009 Edition",
available at http://www.actexmadriver.com/

Present value and discount

Suppose that we make an investment of \$k in an account earning compound interest with effective annual rate of interest i. t years later the balance in this account is $k(1+i)^t$. Here, $k(1+i)^t$ is the future value of the investment t years in the future. Under compound interest, balances multiply by $(1+i)^t$ every t years. $\$k$ at time s is worth $\$k(1+i)^t$ at time $s+t$.

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The quantity $(1+i)$ is called the **interest factor**. $\$k$ at time s is worth $\$k(1+i)$ at time $s+1$.

Often, we need to find the amount of money t years in the past needed to accumulate certain principal. The present value t years in the past is the amount of money which will accumulate to the principal over t years.
In the case of compound interest with effective annual rate of interest i, the present value of \$1 t years in the past is $\frac{1}{(1+i)^t}$. If we invested $\frac{1}{(1+i)^t}$ t years ago in account earning compound interest, then the current balance is \$1. The quantity $\frac{1}{(1+i)^t}$ is called the t **year discount**. $\$k$ at time s is worth $\$k\nu^t$ at time $s-t$.
The quantity $\nu = \frac{1}{1+i}$ is called the **discount factor**. In order to accumulate \$1, we need $\$\nu$ one year in the past.

Under the accumulation function $a(t)$,

- The n–**th year interest factor** is $\frac{a(n)}{a(n-1)}$.
- The effective rate of interest in the n–th year is $i_n = \frac{a(n)-a(n-1)}{a(n-1)}$.
- The n **year discount factor** is $\nu_n = \frac{a(n-1)}{a(n)}$.
- The **effective rate of discount** in the n–th year is $d_n = \frac{a(n)-a(n-1)}{a(n)}$.

i_n and d_n are both proportions of interest over amount values, but i_n uses the amount value in the past and d_n uses the amount value in the future. Since the amount value in the future is bigger than the amount value in the past, $d_n < i_n$.

Notice that

- the n–th year interest factor is equal to $1 + i_n$.
- $\nu_n = 1 - d_n$.
- $1 = (1 + i_n)\nu_n = (1 + i_n)(1 - d_n)$
- $\{1 \text{ unit at time } n - 1\} \equiv \{1 + i_n \text{ units at time } n\}$. So, $d_n = \frac{i_n}{1+i_n}$
- $\{1 - d_n \text{ unit at time } n - 1\} \equiv \{1 \text{ units at time } n\}$. So, $i_n = \frac{d_n}{1-d_n}$.

Under compound interest, the effective rate of discount d_n is constant $d_n = \frac{a(n)-a(n-1)}{a(n)} = \frac{(1+i)^n-(1+i)^{n-1}}{(1+i)^n} = \frac{i}{1+i}$. Under compound interest,

$$\nu = \frac{1}{1+i},\ d = 1-\nu,\ d = \frac{i}{i+1} \ \text{ and } (1-d)(1+i) = 1.$$

Example 1

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Solution: (i)Peter earns an interest amount of $765 - 738 = 27$. The effective annual interest rate earned by Peter is $\frac{27}{738} = 3.658537\%$.
(ii) The effective annual discount rate earned by Peter is $\frac{27}{765} = 3.529412\%$.

Example 2

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Solution: We have that $d = \frac{i}{1+i} = \frac{0.07}{1+0.07} = 6.5421\%$ and $\nu = \frac{1}{1+i} = \frac{1}{1+0.07} = 0.934579$.

Example 3

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Solution: We have that $d = 1 - \nu = 1 - 0.95 = 0.05$ and $i = \frac{1}{\nu} - 1 = \frac{1-0.95}{0.95} = 5.2632\%$.

Example 4

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Solution: The value is $(5000)(1-0.07)^7 = 3008.504353$.

Example 5

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Solution: $(3500)(0.96)^{-2.5} = 3876.055.$

