Manual for SOA Exam FM/CAS Exam 2.

Chapter 1. Basic Interest Theory.

Section 1.5. Nominal rates of interest and discount.

Extract from:
"Arcones' Manual for the SOA Exam FM/CAS Exam 2,
Financial Mathematics. Spring 2009 Edition",
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Nominal rate of interest

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- \$1 at time zero grows to $\$\left(1 + \frac{i^{(m)}}{m}\right)^{mt}$ in t years.
- The accumulation function is $a(t) = \left(1 + \frac{i^{(m)}}{m}\right)^{mt}$.

Example 1

Paul takes a loan of \$569. Interest in the loan is charged using compound interest. One month after a loan is taken the balance in this loan is \$581.
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$$i^{(12)} = (12)(0.02108963093) = 25.30755712\%.$$

Two rates of interest or discount are said to be equivalent if they give rise to same accumulation function. Since, the accumulation function under an annual effective rate of interest i is $a(t) = (1+i)^t$, we have that a nominal annual rate of interest $i^{(m)}$ compounded m times a year is equivalent to an annual effective rate of interest i, if the rates

$$a(t) = \left(1 + \frac{i^{(m)}}{m}\right)^{mt}$$

and

$$a(t) = (1+i)^t$$

agree. This happens if and only if

$$\left(1 + \frac{i^{(m)}}{m}\right)^{m} = 1 + i.$$

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Solution: We find

$$8000\left(1+\frac{0.10}{4}\right)^{\frac{30}{12}\cdot 4} = 8000\,(1+0.025)^{10} = 10240.68.$$

In the calculator, we do:
$\boxed{8000}$ $\boxed{\text{PV}}$ $\boxed{2.5}$ $\boxed{\text{I/Y}}$ $\boxed{10}$ $\boxed{\text{N}}$ $\boxed{\text{CPT}}$ $\boxed{\text{FV}}$.

The calculator TI–BA–II–Plus has a worksheet to convert nominal rates of interest into effective rates of interest and vice versa. To enter this worksheet press $\boxed{\text{2nd}}$ $\boxed{\text{ICONV}}$. There are 3 entries in this worksheet: $\boxed{\text{NOM}}$, $\boxed{\text{EFF}}$ and $\boxed{\text{C/Y}}$. $\boxed{\text{C/Y}}$ is the number of times the nominal interest is converted in a year. The relation between these variables is

$$1 + \frac{\boxed{\text{EFF}}}{100} = \left(1 + \frac{\boxed{\text{NOM}}}{100\boxed{\text{C/Y}}}\right)^{\boxed{\text{C/Y}}}.$$

You can enter a value in any of these entries by moving to that entry using the arrows: $\boxed{\uparrow}$ and $\boxed{\downarrow}$. To enter a value in one entry, type the value and press $\boxed{\text{ENTER}}$. You can compute the corresponding nominal (effective) rate by moving to the entry $\boxed{\text{NOM}}$ ($\boxed{\text{EFF}}$) and pressing the key $\boxed{\text{CPT}}$. It is possible to enter negative values in the entries $\boxed{\text{NOM}}$ and $\boxed{\text{EFF}}$. However, the value in the entry $\boxed{\text{C/Y}}$ has to be positive.

Example 2

If $i^{(4)} = 5\%$ find the equivalent effective annual rate of interest.

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Solution: We solve $1 + i = \left(1 + \frac{0.05}{4}\right)^4$ and get $i = 5.0945\%$. In the calculator, you enter the worksheet [ICONV] and enter: [NOM] equal to 5 and [C/Y] equal to 4. Then, go to [EFF] and press [CPT]. To quit, press [2nd], [QUIT].

Example 3

If $i = 5\%$, what is the equivalent $i^{(4)}$?

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Solution: We solve $\left(1+\frac{i^{(4)}}{4}\right)^4 = 1+0.05$ we get that $i^{(4)} = 4\left((1+0.05)^{1/4}-1\right) = 4.9089\%$. In the calculator, you enter the worksheet ICONV and enter: EFF equal to 5 and C/Y equal to 4. Then, go to NOM and press CPT. To quit, press 2nd, QUIT.

The **nominal rate of discount** $d^{(m)}$ is defined as the value such that 1 unit at the present is equivalent to $1 - \frac{d^{(m)}}{m}$ units invested $\frac{1}{m}$ years ago, i.e.

$$\{1 - \frac{d^{(m)}}{m} \text{ units at time } 0\} \equiv \left\{1 \text{ unit at time } \frac{1}{m}\right\}.$$

This implies that

$$\{1 \text{ unit at time } 0\} \equiv \left\{\frac{1}{1 - \frac{d^{(m)}}{m}} \text{ units at time } \frac{1}{m}\right\}.$$

The accumulation function for compound interest under a the nominal rate of discount $d^{(m)}$ convertible m times a year is $a(t) = \left(1 - \frac{d^{(m)}}{m}\right)^{-mt}$. We have that

$$1 + i = \left(1 + \frac{i^{(m)}}{m}\right)^{m} = (1-d)^{-1} = \left(1 - \frac{d^{(m)}}{m}\right)^{-m}.$$

In the calculator TI–BA–II–Plus, you may:

- given $i^{(m)}$, find i, by entering $i^{(m)} \to$ NOM and $m \to$ C/Y, then in EFF press CPT.
- given i, find $i^{(m)}$, by entering $i \to$ EFF and $m \to$ C/Y, then in NOM press CPT.
- given $d^{(m)}$, find d, by entering $-d^{(m)} \to$ NOM and $m \to$ C/Y, then in EFF press CPT. d appears with a negative sign.
- given d, find $d^{(m)}$, by entering $-d \to$ EFF and $m \to$ C/Y, then in NOM press CPT. $d^{(m)}$ appears with a negative sign.
- given i, find d, by using the formula $i = \frac{1}{1-d} - 1$.
- given d, find i, by using the formula $d = 1 - \frac{1}{1+i}$.

Example 4

If $d^{(4)} = 5\%$ find i.

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Solution: We solve $\left(1 - \frac{d^{(4)}}{4}\right)^{-4} = 1 + i$ to get $d = 4.9070\%$ and $i = 5.1602\%$. In the calculator, in the $\boxed{\text{ICONV}}$ worksheet, we enter -5 in $\boxed{\text{NOM}}$, 4 in $\boxed{\text{C/Y}}$ and we find that $\boxed{\text{EFF}}$ is -4.9070%, then we do
$\boxed{-4.9070}$ $\boxed{\%}$ $\boxed{+}$ $\boxed{1}$ $\boxed{=}$ $\boxed{1/x}$ $\boxed{-}$ $\boxed{1}$ $\boxed{=}$
to get $i = 5.1602\%$.

Example 5

If $i = 3\%$ find $d^{(2)}$.

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Solution: We solve for $d^{(2)}$ in $\left(1 - \frac{d^{(2)}}{2}\right)^{-2} = 1 + i$. First we find that $d = 2.9126\%$ doing

$\boxed{3}$ $\boxed{\%}$ $\boxed{+}$ $\boxed{1}$ $\boxed{=}$ $\boxed{1/x}$ $\boxed{-}$ $\boxed{1}$ $\boxed{=}$

Then, using the ICONV worksheet, we get that $d^{(2)} = 2.9341\%$.

