Manual for SOA Exam FM/CAS Exam 2.

Chapter 2. Cashflows.

Section 2.1. Cashflows.

Extract from:
"Arcones' Manual for the SOA Exam FM/CAS Exam 2,
Financial Mathematics. Fall 2009 Edition",
available at http://www.actexmadriver.com/

Net present value of cash flows

Recall that a **cashflow** is a series of payments made at different times. We can represent a cashflow in a table:

Investments	C_1	C_2	$\cdots$	C_n
Time (in periods)	t_1	t_2	$\cdots$	t_n

Assuming compound interest, the **present value of a cashflow** at time t is

$$V(t) = \sum_{j=1}^{n} C_j \nu^{t_j - t} = \sum_{j=1}^{n} C_j (1+i)^{t-t_j},$$

where i is the effective interest per period and ν is the discount factor per period. The previous equation is the **equation of value**. Under the accumulation function $a(\cdot)$, the equation of value is

$$V(t) = \sum_{j=1}^{n} C_j \frac{a(t)}{a(t_j)}.$$

Example 1

An investor can invest in a project which requires an investment of \$37400 at time 0. The investment pays \$25000 at time 1 and \$15000 at time 2. The investor's capital is currently earning an effective annual rate of interest of 4.5%. Should the investor invest in the project?

Example 1

An investor can invest in a project which requires an investment of \$37400 at time 0. The investment pays \$25000 at time 1 and \$15000 at time 2. The investor's capital is currently earning an effective annual rate of interest of 4.5%. Should the investor invest in the project?

Solution: The net present value of the investment in the project is

$$\begin{aligned} &25000(1.045)^{-1} + 15000(1.045)^{-2} - 37400 \\ =&23923.445 + 13735.9493 - 37400 = 259.3943. \end{aligned}$$

Yes, the investor should invest in the project.

A way to analyze the profitability of an investment is to find the present value at time of the cashflow derived from the investment. The **net present value** of an investment is the present value of the inflows minus the present value of the outflows.
Suppose that a company can select between taking two investments. Which investment has the biggest net present value depends on the used interest rate. To valuate investments, companies use their **cost of capital**. The cost of capital of a company is an estimation of how much the company has to pay for every dollar it borrows. This cost of capital is found using the whole capital components of the company.

Example 2

A company has cost of capital of 7.5% as an annual effective rate of interest. Two investment projects have the following forecasted cash flows:

Project A	−$20,000	0	0	$25,000	$10,000
Project B	−$20,000	0	0	$10,000	$26,000
Time in years	0	1	2	3	4

(i) Find the profit made under each investment project.
(ii) Which project has the highest profit?
(iii) Compute the net present value for each project using the company's cost of capital.
(iv) Which project has the highest net present value?

Solution:(i) The profit for project A is

$$-20000 + (25000) + (10000) = 15000.$$

The profit for project B is

$$-20000 + (10000) + (26000) = 16000.$$

(ii) Project B has the highest profit.
(iii) The present value of project A is

$$-20000 + (25000)(1.075)^{-3} + (10000)(1.075)^{-4} = 7612.02.$$

The present value of project B is

$$-20000 + (10000)(1.075)^{-3} + (26000)(1.075)^{-4} = 7518.419.$$

(iv) Project A has the highest net present value.

The calculator TI–BA–II–Plus has a **cashflow worksheet**, which allows to work with cashflows when the deposit times are nonnegative numbers. After entering the cashflow in the calculator, you can find either **the present value** of the cashflow or the **internal rate of return**. The internal rate of return is the effective periodic rate of interest. There are 3 keys to enter different parts of this worksheet.

- Pressing the key CF, you can enter the cash flow data.
- Pressing the key NPV opens a worksheet with two variables NPV and I. Using this worksheet, you can compute the net present value.
- Pressing the IRR you compute the internal rate of return.

When you press the key [CF], you can change the entries: [CFo], [C01], [F01], [C02], [F02], ..., [C24], [F24]. [CFo] is the initial contribution. [C01] is the amount of the first round of contributions. [F01] is the (number of payments) frequency of the first round of contributions. [C02] is the amount of the second round of contributions. [F02] is the frequency of the second round of contributions. [C03], [F03], ..., [C24], [F24] are defined similarly. To move from one entry to the next, use the arrows [↓] and [↑]. To enter a number in one entry, type the number, and then press [ENTER]. To clear the values in the worksheet, press [2nd] [CLR WORK] while in the worksheet. To exit the worksheet type [2nd] [QUIT]. The cashflow, which we have is:

Payments	CFo	C01	C02	...
Time	0	1 to F01	F01+1 to F01+F02	...

After you have entered the date you can calculate either the net present value or the internal rate of return.

- To calculate the net present value, press the key [NPV], enter the periodic interest rate in the entry [I], then go the entry [NPV] using one of the arrows [↓] and [↑]. Finally press [CPT].
- To calculate the internal rate of return, press the keys [IRR] and [CPT]. If the equation does not have a solution, you get "Error 5". If the equation has several solutions, you get the one with smallest absolute value.

Example 3

Joel wishes to borrow a sum of money. In return, he is prepared to pay as follows: \$100 after 1 year, \$200 after 2 years, \$300 after 3 years, and \$400 after 4 years. If $i = 12\%$, how much can he borrow?

Example 3

Joel wishes to borrow a sum of money. In return, he is prepared to pay as follows: \$100 after 1 year, \$200 after 2 years, \$300 after 3 years, and \$400 after 4 years. If $i = 12\%$, how much can he borrow?

Solution: The cashflow is

contributions	100	200	300	400
Time	1	2	3	4

He can borrow:

$$(100)(1+0.12)^{-1} + (200)(1+0.12)^{-2}$$
$$+(300)(1+0.12)^{-3} + (400)(1+0.12)^{-4} = 716.4657955$$

To do this problem using the CF worksheet. Press CF, and enter CFo=0, C01=100, F01=1, C02=200, F02=1, C03=300, F03=1, C04=400, F04=1, 2nd QUIT. Go to NPV, enter I=12, and compute NPV and get 716.4657955.

Example 4

A loan with an effective annual interest rate of 5.5% is to be repaid with the following payments:
(i) 1000 at the end of the first year.
(ii) 2000 at the end of the second year.
(iii) 5000 at the end of the third year.
Calculate the loaned amount at time 0.

Example 4

A loan with an effective annual interest rate of 5.5% is to be repaid with the following payments:
(i) 1000 at the end of the first year.
(ii) 2000 at the end of the second year.
(iii) 5000 at the end of the third year.
Calculate the loaned amount at time 0.

Solution: The cashflow of payments to the loan is

Payments	1000	2000	5000
Time	1	2	3

The loaned amount at time zero is the present value at time zero of the cashflow of payments, which is

$$(1000)(1.055)^{-1} + (2000)(1.055)^{-2} + (5000)(1.055)^{-3} = 7002.840451.$$

Press [CF], and enter [CFo]=0, [C01]=1000, [F01]=1, [C02]=2000, [F02]=1, [C03]=5000, [F03]=1, [2nd] [QUIT]. Go to [NPV], enter [I]=5.5, and compute [NPV] and get 7002.840451.

Example 5

Helen borrows \$5000 from her credit card account at a nominal annual interest rate of 20% per year convertible monthly. Two months later, she pays \$1000 back. Four months after the payment she borrows \$2000. How much does she owe one year after the loan is taken out?

Solution: The cashflow is

inflow/outflow	5000	0	−1000	0	0	0	2000
Time (in months)	0	1	2	3	4	5	6

Example 5

Helen borrows \$5000 from her credit card account at a nominal annual interest rate of 20% per year convertible monthly. Two months later, she pays \$1000 back. Four months after the payment she borrows \$2000. How much does she owe one year after the loan is taken out?

Solution: The cashflow is

inflow/outflow	5000	0	−1000	0	0	0	2000
Time (in months)	0	1	2	3	4	5	6

Example 5

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Solution: The cashflow is

inflow/outflow	5000	0	−1000	0	0	0	2000
Time (in months)	0	1	2	3	4	5	6

The equation of value at time 1 year is

$$5000\left(1+\frac{0.20}{12}\right)^{12}-1000\left(1+\frac{0.20}{12}\right)^{10}+2000\left(1+\frac{0.20}{12}\right)^{6}$$

$$=7125.737519$$

Example 5

Helen borrows \$5000 from her credit card account at a nominal annual interest rate of 20% per year convertible monthly. Two months later, she pays \$1000 back. Four months after the payment she borrows \$2000. How much does she owe one year after the loan is taken out?

Solution: The cashflow is

inflow/outflow	5000	0	−1000	0	0	0	2000
Time (in months)	0	1	2	3	4	5	6

Press [CF], and enter [CFo]=5000, [C01]=0, [F01]=1, [C02]=−1000, [F02]=1, [C03]=0, [F03]=3, [C04]=2000, [F04]=1, [2nd] [QUIT]. Go to [NPV], enter [I]=1.66666(=20/12), and compute [NPV]= 5843.69. This is the present value at time 0 of the loan. The future value of the loan at time 1 year is $5843.69(1.01666)^{12} = 7125.737519$.

