Manual for SOA Exam FM/CAS Exam 2.

Chapter 2. Cashflows.

Section 2.2. Method of equated time.

Extract from:
"Arcones' Manual for the SOA Exam FM/CAS Exam 2,
Financial Mathematics. Fall 2009 Edition",
available at http://www.actexmadriver.com/

Given the cashflow

Investments	C_1	C_2	$\cdots$	C_n
Time	t_1	t_2	$\cdots$	t_n

we would like to find a time $\tilde{t}$ such that a lump sum $C = \sum_{j=1}^{n} C_j$ invested at time $\tilde{t}$ is equivalent to the previous cashflow. To find $\tilde{t}$, we solve the equation,

$$C\nu^{\tilde{t}} = \sum_{j=1}^{n} C_j \nu^{t_j}, \tag{1}$$

and get

$$\tilde{t} = \frac{\ln(\sum_{j=1}^{n} C_j \nu^{t_j}/C)}{\ln \nu} \tag{2}$$

Method of equated time

The **method of equated time** consists on approximating $\tilde{t}$ by

$$\overline{t} = \frac{\sum_{j=1}^{n} C_j t_j}{C} = \frac{\sum_{j=1}^{n} C_j t_j}{\sum_{j=1}^{n} C_j}. \tag{3}$$

This is the average time of all the times t_j with the weight $\frac{C_j}{\sum_{k=1}^{n} C_k}$ at t_j.

The first order Taylor expansion of $\nu^t = (1+i)^{-t}$ on i is $1 - ti$.
So, Equation (1) is approximately

$$C(1 - ti) = \sum_{j=1}^{n} C_j(1 - t_j i),$$

whose solution is

$$\bar{t} = \frac{\sum_{j=1}^{n} C_j t_j}{C} = \frac{\sum_{j=1}^{n} C_j t_j}{\sum_{j=1}^{n} C_j}.$$

If the interest were simple, the future value at time t_n of the considered cashflow would be

$$C(1 + (t_n - t)i) = \sum_{j=1}^{n} C_j(1 + (t_n - t_j)i),$$

whose solution is $t = \frac{\sum_{j=1}^{n} C_j t_j}{C}$.
The approximation to t using the method of equating time is the solution to the considered problem when the accumulation function follows simple interest.

Example 1

Payments of \$300, \$100 and \$200 are due at the ends of years 1, 3, and 5, respectively. Assume an annual effective rate of interest of 5% per year. (i) Find the point in time at which a payment of \$600 would be equivalent. (ii) Find the approximation to this point using the method of equated time.

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Solution: (i) The time $\tilde{t}$ solves the equation

$$600\nu^{\tilde{t}} = 300\nu + 100\nu^3 + 200\nu^5$$
$$= 285.71429 + 86.38376 + 156.70523 = 528.80328,$$

where $\nu = (1.05)^{-1}$. Hence, $\tilde{t} = \frac{\ln(600/528.80328)}{\ln(1.05)} = 2.58891$.

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where $\nu = (1.05)^{-1}$. Hence, $\tilde{t} = \frac{\ln(600/528.80328)}{\ln(1.05)} = 2.58891$.
(ii) The equated time approximation to this point is

$$\bar{t} = \frac{(300)(1) + (100)(3) + (200)(5)}{600} = 2.666667.$$

