

Manual for SOA Exam FM/CAS Exam 2.

Chapter 2. Cashflows.

Section 2.5. Investment and portfolio methods.

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Investment and portfolio methods

Suppose that an investment fund pools money from several identities (individuals or corporations) and makes investments on behalf of them. Then, the fund faces the question: how to allocate the returns between the different identities? There are two main ways to allocate interest to the various accounts: the **portfolio method** and the **investment year method**.

The portfolio method is an accounting method that credits all funds one specified current rate of interest, regardless of when the money was placed in the account. Usually this rate of interest changes from year to year. Let i^y denote the annual interest rate credited in year y . If x is invested at the beginning of the year y , then the balance in the account in the year $y + t$ is

$$x \prod_{j=0}^{t-1} (1 + i^{y+j}) = x(1 + i^y)(1 + i^{y+1}) \cdots (1 + i^{y+t-1}).$$

Example 1

Suppose that an investment account credits investors using the portfolio method with the annual rates in the following table:

| <i>Calendar year of portfolio rate</i> | <i>Portfolio rates</i> |
|--|----------------------------|
| y | i^y |
| 1999 | 4.50% |
| 2000 | 5.50% |
| 2001 | 4.00% |
| 2002 | 6.50% |

Suppose that 100 was invested on January 1, 1999.

- (i) Find the balance on January 1, 2000.
- (ii) The balance on January 1, 2001.
- (iii) The balance on July 1, 2001.

Solution: (i) The balance on January 1, 2000, is
 $(100)(1.045) = 104.5$.

(ii) The balance on January 1, 2001, is
 $(100)(1.045)(1.055) = 110.2475$.

(iii) The balance on July 1, 2001, is

$$(100)(1.045)(1.055)(1.04)^{1/2} = 112.4308.$$

The investment year method is an accounting method in which an investment fund keeps records of the interest rates it earns annually on funds assigned each year to accounts within the general account. The investment year method is also called the new money method. We will assume that accounts are made according with the year at which the money was invested. For example, suppose that an investment account credits investors according with the investment year method using the following table:

| Calendar year of original investment | Investment year rates | | | | |
|--------------------------------------|-----------------------|---------|---------|---------|---------|
| y | i_1^y | i_2^y | i_3^y | i_4^y | i_5^y |
| 1999 | 4.25% | 4.35% | 4.47% | 4.57% | 4.70% |
| 2000 | 4.56% | 4.73% | 4.75% | 4.98% | 4.04% |
| 2001 | 4.05% | 4.04% | 4.13% | 4.17% | 4.24% |
| 2002 | 4.45% | 4.15% | 4.23% | 4.36% | 4.44% |
| 2003 | 4.25% | 4.35% | 4.55% | 9.55% | 5.65% |

| Calendar year of original investment | Investment year rates | | | | |
|--------------------------------------|-----------------------|---------|---------|---------|---------|
| y | i_1^y | i_2^y | i_3^y | i_4^y | i_5^y |
| 1999 | 4.25% | 4.35% | 4.47% | 4.57% | 4.70% |
| 2000 | 4.56% | 4.73% | 4.75% | 4.98% | 4.04% |
| 2001 | 4.05% | 4.04% | 4.13% | 4.17% | 4.24% |
| 2002 | 4.45% | 4.15% | 4.23% | 4.36% | 4.44% |
| 2003 | 4.25% | 4.35% | 4.55% | 9.55% | 5.65% |

This means that money invested during 2000 earns an effective rate of interest of 4.56% during 2000, it earns an effective rate of interest of 4.73% during 2001, and so on. For example, if an account is open with an investment of x invested on January 1, 2000, then:

the balance on January 1, 2001 is $(100)(1.0456)$;

the balance on January 1, 2002 is $(100)(1.0456)(1.0473)$; and so on.

Example 2

An investment fund applies the investment year method for the first two years, after which a portfolio rate is used. The following table of interest rates is used:

| <i>Calendar year of original investment</i> | <i>Investment year rates</i> | | <i>Portfolio rates</i> |
|---|----------------------------------|---------|----------------------------|
| y | i_1^y | i_2^y | i^{y+2} |
| 2000 | 5.25% | 5.25% | 5.40% |
| 2001 | 5.35% | 5.35% | 5.65% |
| 2002 | 5.45% | 5.45% | 5.10% |
| 2003 | 5.45% | 5.45% | 5.34% |
| 2004 | 5.50% | 5.35% | 5.55% |
| 2005 | 5.50% | 5.55% | 5.65% |

(i) Ashley invests \$1000 into the fund on January 1, 2000. The investment year method is applicable for the first two years, after which a portfolio rate is used. Calculate Ashley's account accumulation on January 1, 2006.

(ii) Elizabeth invests \$1000 into the fund on January 1, 2000. But, she redeemed her investment from the fund at the end of every year and reinvested the money at the new money rate. Calculate Elizabeth's accumulation account on January 1, 2006.

Solution: (i) Ashley's account accumulation on January 1, 2006 is

$$(1000)(1.0525)(1.0525)(1.0540)(1.0565)(1.0510)(1.0534) = 1365.684.$$

(ii) Elizabeth's investment value on January 1, 2006 is

$$(1000)(1.0525)(1.0535)(1.0545)(1.0545)(1.055)(1.050) = 1365.814.$$