

Manual for SOA Exam FM/CAS Exam 2.
Chapter 4. Amortization and sinking bonds.
Section 4.3. Reinvestment.

©2009. Miguel A. Arcones. All rights reserved.

Extract from:

"Arcones' Manual for the SOA Exam FM/CAS Exam 2,
Financial Mathematics. Fall 2009 Edition",
available at <http://www.actexamdriver.com/>

Suppose that a bank account pays interest in the original deposit, but not in the obtained interest. Then, it will be wise to withdraw the interest and invest it in another account. In other situations, it makes sense to reinvest the earned interest in a different investment. For example, a stock pays dividends and/or capital gains, which can be invested somewhere else. We obtain a flow of interest payments which are invested at a different rate from the one in the initial investment. Suppose that a mortgage company makes a loan to a customer. To know the mortgage company's return in its investment we need to take into account the interest rate charged to the customer in the loan and the interest rate which the mortgage company gets in the monthly payments it receives.

Example 1

Payments of \$2500 are invested at the end of each quarter for 5 years. The payments earn interest at an annual nominal interest rate converted quarterly of 14% and the interest payments are reinvested at an annual nominal interest rate converted quarterly of 10%. Find the total accumulation for both accounts at the end of 5 years.

Solution: The balance in the first account is

Balance in the 1st account	2500	2(2500)	(20)(2500)
Time (in quarters)	1	2	20

Since the rate of interest per quarter is $14\%/4 = 3.5\%$, the interest payments are

Cashflow of interest	(1)(0.035)(2500)	...	19(0.035)(2500)
Time (in quarters)	2	...	20

The total accumulation for both accounts at the end of the 5-th year is

$$\begin{aligned}
 & (20)(2500) + (0.035)(2500) (Is)_{19|0.025} \\
 = & 50000 + (87.5) \left(\frac{(1.025)^{19} - 1}{0.025} \right) \\
 = & 50000 + (87.5) \left(\frac{24.54465761 - 1}{0.025} \right) = 69406.30164.
 \end{aligned}$$

Example 2

John invests 1000 at the beginning of each year for 5 years at an annual effective interest rate of 10% and reinvests the interest at an annual effective interest rate of 8%. Calculate the total value of his investment at the end of 5 years.

Solution: The balance in John's account is

Balance in John's account	1000	2000	3000	4000	5000	5000
Time	0	1	2	3	4	5

The interest payments which John gets are

John's interest payments	0	100	200	300	400	500
Time	0	1	2	3	4	5

The total accumulation of John's investments at time 5 is

$$\begin{aligned}
 5000 + 100 (Is)_{\overline{5}|0.08} &= 5000 + 100 \left(\frac{(1.08)s_{\overline{5}|0.08} - 5}{0.08} \right) \\
 &= 5000 + 100 \left(\frac{6.335929037 - 5}{0.08} \right) = 6669.911296.
 \end{aligned}$$