Manual for SOA Exam FM/CAS Exam 2.

Chapter 5. Bonds.

Section 5.2. Price of a bond.

Extract from:
"Arcones' Manual for the SOA Exam FM/CAS Exam 2,
Financial Mathematics. Fall 2009 Edition",
available at http://www.actexmadriver.com/

Bonds

There are two kind of bonds: **accumulation bonds** and **bonds with coupons**. The time at which the loan is repaid is called the **maturity date** (or **redemption date**).

- In the case of accumulation bonds, the borrower agrees to pay the loan plus interest at a unique date, called the redemption time. An accumulation bond is also called a **zero coupon bond**.
- The most common bonds are bonds with coupons. For bonds with coupons, the borrower agrees to make period payments (**coupons**) plus a **balloon payment** (the **redemption value**) C at the **maturity date**.

Cashflow of a bond with coupons.

Every bond has a **face value** (or **par value**) F. The coupon payment is Fr. Here, r is the **coupon rate** per interest period. Often, the payments are semiannually and $2r$ is the annual nominal rate of interest convertibly semiannually. A bond is called **redeemable at par** if $C = F$. Unless said otherwise we assume that a bond is redeemable at par. Let n be the number of interest periods until the redemption date. Let i be the yield rate per interest period. The cashflow for the borrower is

Contributions	P	$-Fr$	$-Fr$	$-Fr$	$\cdots$	$-Fr-C$
Time	0	1	2	3	$\cdots$	n

Variables for a bond.

- P = the price of a bond.
- F = the par or face value of a bond.
- C = the redemption value of a bond.
- r = the coupon rate of a bond.
- Fr = the amount of a coupon.
- i = the yield rate of the bond per coupon period.
- $\nu = \frac{1}{1+i}$ = the discount factor per coupon period.
- n = the number of coupon payment periods.
- $g = \frac{Fr}{C}$ = the modified coupon rate of a bond
- $G = \frac{Fr}{i}$ = the base amount of a bond.
- $K = C\nu^n$ = the present value, compounded at the yield rate, of the redemption value of a bond
- $P - C$ = the premium (if $P > C$).
- $C - P$ = the discount (if $C > P$).
- $k = \frac{P-C}{C}$ premium as a fraction of redemption value.

Price of a bond

The **basic formula for the price of a bond** is

$$P = Fra_{\overline{n}|i} + C(1+i)^{-n} = Fra_{\overline{n}|i} + K.$$

The **premium/discount formula for the price of a bond** is

$$P = Fra_{\overline{n}|i} + C\nu^n = Fra_{\overline{n}|i} + C(1 - ia_{\overline{n}|i})$$
$$= C + (Fr - Ci)a_{\overline{n}|i} = C + C(g-i)a_{\overline{n}|i}.$$

The **base amount formula for the price of a bond** is

$$P = Fra_{\overline{n}|i} + C\nu^n = Gia_{\overline{n}|i} + C\nu^n = G(1-\nu^n) + C\nu^n = G + (C-G)\nu^n.$$

The **Makehan formula for the price of a bond** is

$$P = Fra_{\overline{n}|i} + C\nu^n = Cg\frac{1-\nu^n}{i} + C\nu^n$$
$$= \frac{g}{i}(C - C\nu^n) + C\nu^n = \frac{g}{i}(C-K) + K.$$

Example 1

Find the price of a 10–year bond, redeemable at par, with face value of \$10,000 and coupon rate of 10%, convertible quarterly, that will yield 8%, convertible quarterly.

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Solution: We know that $F = C = 10000$, $n = (10)(4) = 40$, $r = \frac{0.10}{4} = 0.025$, $Fr = (10000)(0.025) = 250$. and $i = \frac{0.08}{4} = 0.02$. So, the price of the bond is

$$P = Fra_{\overline{n}|i} + C(1+i)^{-n} = 250a_{\overline{40}|0.02} + 10000(1.02)^{-40}$$

$$=11367.77396.$$

In the calculator, do:

40 | N | 2 | I/Y | 250 | PMT | 10000 | PMT | CPT | PV.

Example 2

A 30 year bond matures at its face value of 10,000. It pays semiannual coupons of 600. Calculate the price of the bond if the annual nominal interest rate convertible semiannually is 7.5%.

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Solution: We know that $F = C = 10000$, $n = (30)(2) = 60$, $Fr = 600$, and $i = \frac{7.5\%}{2} = 3.75\%$. The price of the bond is

$$(600)a_{\overline{60}|0.0375} + (10000)(1 + 0.0375)^{-60} = 15341.03109.$$

Remember that unless said otherwise a bond is redeemable at par.

Example 3

What is the price of a 5–year 100 par–value bond having quarterly coupons at a quarter rate of 1.5% that is bought to yield a nominal annual rate of 12% convertible monthly?

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What is the price of a 5–year 100 par–value bond having quarterly coupons at a quarter rate of 1.5% that is bought to yield a nominal annual rate of 12% convertible monthly?

Solution: **Solution:** We know that $F = C = 100$, $n = (5)(4) = 20$, $r = 0.015$ and $Fr = (100)(0.015) = 1.5$. Let j be the effective yield rate per quarter. We have that $i^{(12)} = 12\%$, $i = 12.68250301\%$ and $i^{(4)} = 12.1204\%$, $j = i^{(4)}/4 = 3.0301\%$. Hence,

$$P = 1.5a_{\overline{20}|3.0301\%} + 100(1.030301)^{-20} = 77.29919664.$$

Example 4

The price of a zero coupon 1000 face value bond is 599.4584. The yield rate convertible semi–annually is 6.5%. Calculate the maturity date.

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Solution: Let n be the maturity date in years. We have that $599.4584 = (1000)\left(1 + \frac{0.065}{2}\right)^{2n}$ and $n = 8$ years.

Example 5

What is the yield as an annual effective rate of interest on a 100 par–value 10–year bond with coupon rate 6%, convertible monthly, that is selling for 90?

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What is the yield as an annual effective rate of interest on a 100 par–value 10–year bond with coupon rate 6%, convertible monthly, that is selling for 90?

Solution: We know that $P = 90$, $C = F = 100$, $n = (10)(12) = 120$, $r = \frac{0.06}{12} = 0.005$ and $Fr = (100)(0.005) = 0.5$. Let $j = i^{(12)}/12$. Then,

$$90 = P = Fra_{\overline{n}|j} + C(1+j)^{-12n} = (0.5)a_{\overline{120}|j} + 100(1+j)^{-120}.$$

Hence, $j = 0.618181404\%$, $i^{(12)} = 12j = 7.419376846\%$ and $i = 7.676949087\%$.

Example 6

A 1000 par value 10–year bond with semiannual coupons and redeemable at 1200 is purchased to yield 8% convertible semiannually. The first coupon is 50*. Each subsequent coupon is 3% greater than the preceding coupon. Find the price of the bond.*

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A 1000 par value 10–year bond with semiannual coupons and redeemable at 1200 is purchased to yield 8% convertible semiannually. The first coupon is 50*. Each subsequent coupon is 3% greater than the preceding coupon. Find the price of the bond.*

Solution: The cashflow of coupons is

Coupons	50	(50)(1.03)	$\cdots$	$(50)(1.03)^{19}$
Time (in half–years)	1	2	$\cdots$	20

The present value of the payments is

$$(50)\frac{1}{1.03}a_{\overline{20}|\frac{0.04-0.03}{1.03}} + (1200)(1.04)^{-20}$$
$$=878.5721 + 547.6643 = 1426.2364.$$

