Manual for SOA Exam FM/CAS Exam 2.

Chapter 5. Bonds.

Section 5.4. Book value between coupon payments dates.

Extract from:
"Arcones' Manual for the SOA Exam FM/CAS Exam 2,
Financial Mathematics. Fall 2009 Edition",
available at http://www.actexmadriver.com/

Book value between coupon payments dates

We know that the book value of a bond immediately after a coupon payment is given by

$$B_k = Fra_{\overline{n-k}|i} + C\nu^{n-k} = C + (Fr - Ci)a_{\overline{n-k}|i} = C + C(g - i)a_{\overline{n-k}|i}.$$

We want to determine the book value of a bond between successive coupon dates. We use as a unit of time coupon periods. We want to determine the value of a bond at time $k + t$ periods, where k is an integer and $0 \leq t < 1$. The present value at time $k + t$ of the remaining payments is $B_k(1 + i)^t$. Immediately before the payment of the $(k + 1)$–th coupon, the price of the bond is $B_k(1 + i)$. Immediately after the payment of the $(k + 1)$–th coupon, the price of the bond is $B_k(1 + i) - Fr$. Hence, the price of a bond is a discontinuous function (see Figure 1).

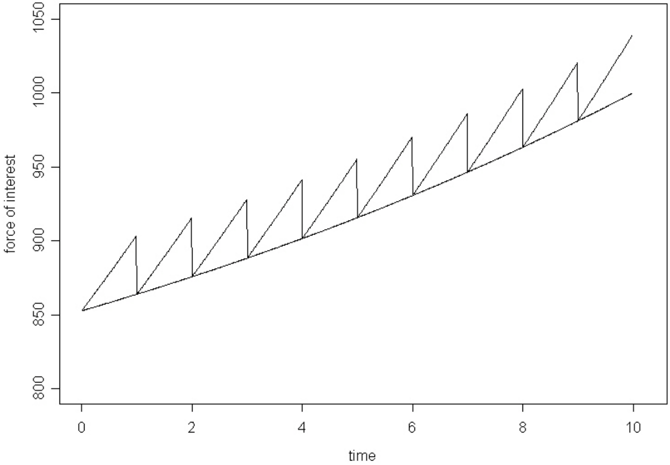

Figure 1: Theoretical flat and market values

During the time period $(k, k+1)$ a bond accrues a coupon payment. The accrued value at time $t \in [0,1)$ of the next coupon is denoted by Fr_t. We have that $Fr_0 = 0$, $\lim_{t\to 1-} Fr_t = Fr$. The **flat price** of a bond is the money that actually changes hands at the date of sale. The **market price** B^m_{k+t} is the price of a bond excluding the accrued value of the next coupon. Hence, we have that $B^f_{k+t} = B^m_{k+t} + Fr_t$.
The flat price (also known as the dirty price) is the book value of a bond. It is the price that an investor pays for a bond. The market price (also known as the clean price) is the price of bond quoted in a newspaper.

There are three methods to determine the flat value, accrued coupon and market value of a bond:

- **Theoretical method:**

$$B^f_{k+t} = B_k(1+i)^t, \; Fr_t = Fr\left(\frac{(1+i)^t - 1}{i}\right)$$
$$B^m_{k+t} = B_k(1+i)^t - Fr\left(\frac{(1+i)^t - 1}{i}\right).$$

- **Practical method:**

$$B^f_{k+t} = B_k(1+ti), \; Fr_t = tFr, \; B^m_{k+t} = B_k(1+ti) - tFr$$

- **Semi–theoretical method:**

$$B^f_{k+t} = B_k(1+i)^t, \; Fr_t = tFr, \; B^m_{k+t} = B_k(1+i)^t - tFr$$

The practical method assumes that the flat price accrues under simple interest. It assumes that the accrued coupon is proportional to the time since the last coupon payment.
As to the theoretical method, $B_k(1+i)^t$ is the present value of the payments to be made. This is actual outstanding balance in the loan at time $k+t$. We have that

$$B^m_{k+t} = B_k(1+i)^t - Fr\left(\frac{(1+i)^t - 1}{i}\right)$$

$$= \left(Fr\left(\frac{1-\nu^{n-k}}{i}\right) + C\nu^{n-k}\right)\nu^{-t} - Fr\left(\frac{\nu^{-t}-1}{i}\right)$$

$$= Fr\left(\frac{1-\nu^{n-k-t}}{i}\right) + C\nu^{n-k-t} = Fra_{\overline{n-k-t}|i} + C\nu^{n-k-t},$$

where $a_{\overline{s}|i} = \frac{1-\nu^s}{i}$, $s > 0$ and s is not necessarily a positive integer.
The market price according with the theoretical method has a continuous function.

Example 1

Find the flat price, the accrued interest and the market price of a 1000 10–year bond with 4% annual coupons, bought to yield 3%, four months after the second coupon has been issued. Use all three methods.

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Find the flat price, the accrued interest and the market price of a 1000 10–year bond with 4% annual coupons, bought to yield 3%, four months after the second coupon has been issued. Use all three methods.

Solution: We have that $F = 1000$, $n = 10$, $r = 0.04$, $Fr = 40$, $i = 3\%$, $k = 2$ and $t = \frac{1}{3}$. So,

$$B_2 = Fra_{\overline{n-k}|i} + C\nu^{n-k} = 40a_{\overline{8}|3\%} + (1000)(1.03)^{-8} = 1070.20.$$

Using the theoretical method,

$$B^f_{2+(1/3)} = B_2(1+i)^{1/3} = (1070.20)(1.03)^{1/3} = 1080.80,$$

$$Fr_{(1/3)} = Fr\left[\frac{(1+i)^{1/3}-1}{i}\right] = (40)\left[\frac{(1+0.03)^{1/3}-1}{0.03}\right] = 13.20$$

$$B^m_{2+(1/3)} = B^f_{2+(1/3)} - Fr_{(1/3)} = 1080.80 - 13.20 = 1067.60.$$

Example 1

Find the flat price, the accrued interest and the market price of a 1000 10–year bond with 4% annual coupons, bought to yield 3%, four months after the second coupon has been issued. Use all three methods.

Solution: We have that $F = 1000$, $n = 10$, $r = 0.04$, $Fr = 40$, $i = 3\%$, $k = 2$ and $t = \frac{1}{3}$. So,

$$B_2 = Fra_{\overline{n-k}|i} + C\nu^{n-k} = 40a_{\overline{8}|3\%} + (1000)(1.03)^{-8} = 1070.20.$$

Using the practical method,

$$B^f_{2+(1/3)} = B_2(1 + (1/3)i) = (1070.20)(1 + (0.03)/3)) = 1080.90,$$
$$Fr_{(1/3)} = (1/3)Fr = (1/3)(40) = 13.33$$
$$B^m_{2+(1/3)} = B^f_{2+(1/3)} - Fr_{(1/3)} = 1080.90 - 13.33 = 1067.57$$

Example 1

Find the flat price, the accrued interest and the market price of a 1000 10–year bond with 4% annual coupons, bought to yield 3%, four months after the second coupon has been issued. Use all three methods.

Solution: We have that $F = 1000$, $n = 10$, $r = 0.04$, $Fr = 40$, $i = 3\%$, $k = 2$ and $t = \frac{1}{3}$. So,

$$B_2 = Fra_{\overline{n-k}|i} + C\nu^{n-k} = 40a_{\overline{8}|3\%} + (1000)(1.03)^{-8} = 1070.20.$$

Using the semi–theoretical method:

$$B^f_{2+(1/3)} = B_2(1+i)^{1/3} = (1070.20)(1.03)^{1/3} = 1080.80,$$
$$Fr_{(1/3)} = (1/3)Fr = (1/3)(40) = 13.33,$$
$$B^m_{2+(1/3)} = B^f_{2+(1/3)} - Fr_{(1/3)} = 1080.79 - 13.33 = 1067.46.$$

