# Manual for SOA Exam FM/CAS Exam 2.

## Chapter 5. Bonds.
## Section 5.5. Callable bonds.



Extract from:
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# Callable bonds

A **callable bond** is a bond which gives the issuer (not the investor) the right to redeem prior to its maturity date, under certain conditions. When issued, the **call provisions** explain when the bond can be redeemed and what the price will be. In most cases, there is some period of time during which the bond cannot be called. This period of time is named the **call protection period**. The earliest time to call the bond is named the **call date**. The **call price** is the amount of money the insurer must pay to buy the bond back.

Usually, bonds can be only called immediately after the payment of a coupon. We will study the computation of the yield rate of return for the investor in this situation. Since the investor does not know the cashflow obtained from his investment, he will assume that the issuer calls the bond under the worst possible situation (in the sense of lowest possible interest rates).



 

If the bond is called immediately after the payment of $k$–th coupon, the present value of the obtained payments is

$$P_k = Fra_{\overline{k|}i} + C\nu^k = C + (Fr - Ci)a_{\overline{k|}i} = C + C(g - i)a_{\overline{k|}i}.$$

$P_k$ is the price which the investor would pay for the bond assuming that the bond is called immediately after the $k$ coupon. As smaller as $P_k$ is, as worst for the lender is. Between all possible choices to recall a bond, the borrower will choose the option with the smallest price. Assuming that the redemption value is a constant and that a bond can be called after any coupon payment:
(i) if $Fr > Ci$ (bond sells at a premium), $P_k$ increases with $k$, and we assume the redemption date is the earliest possible.
(ii) if $Fr < Ci$ (bond sells at a discount), $P_k$ decreases with $k$, and we assume that the redemption date is the latest possible.
If one investor wants to get an effective rate of interest of $i$ per period, then the maximum price which the investor should pay is the lowest possible $P_k$ under that particular rate $i$.



 

## Example 1

*Consider a 100 par–value 8% bond with semiannual coupons callable at 120 on any coupon date starting 5 years after issue for the next 5 years, at 110 starting 10 years after issue for the next 5 years and maturing at 105 at the end of 15 years. What is the highest price which an investor can pay and still be certain of a yield of 9% converted semiannually.*



 

## Example 1

*Consider a 100 par–value 8% bond with semiannual coupons callable at 120 on any coupon date starting 5 years after issue for the next 5 years, at 110 starting 10 years after issue for the next 5 years and maturing at 105 at the end of 15 years. What is the highest price which an investor can pay and still be certain of a yield of 9% converted semiannually.*

**Solution:** We have that $F = 100$, $r = 4\%$, $Fr = 4$, $n = 30$ and $i = 4.5\%$. Let $k$ be the number of the half year when the bond is called.

If $10 \le k \le 19$, then $C = 120$ and $Ci = (120)(0.045) = 5.4$. So, the bond sells at a discount. We have that lowest price which the investor can get is

$$P_{19} = 4a_{\overline{19}|4.5\%} + 120(1.045)^{-19} = 102.37.$$



 

## Example 1

*Consider a 100 par–value 8% bond with semiannual coupons callable at 120 on any coupon date starting 5 years after issue for the next 5 years, at 110 starting 10 years after issue for the next 5 years and maturing at 105 at the end of 15 years. What is the highest price which an investor can pay and still be certain of a yield of 9% converted semiannually.*

**Solution:** We have that $F = 100$, $r = 4\%$, $Fr = 4$, $n = 30$ and $i = 4.5\%$. Let $k$ be the number of the half year when the bond is called.

If $20 \le k \le 29$, then $C = 110$ and $Ci = (110)(0.045) = 4.95$. So, the bond sells at a discount. We have that lowest price which the investor can get is

$$P_{29} = 4a_{\overline{29}|4.5\%} + (110)(1.045)^{-29} = 94.78.$$



 

## Example 1

*Consider a 100 par–value 8% bond with semiannual coupons callable at 120 on any coupon date starting 5 years after issue for the next 5 years, at 110 starting 10 years after issue for the next 5 years and maturing at 105 at the end of 15 years. What is the highest price which an investor can pay and still be certain of a yield of 9% converted semiannually.*

**Solution:** We have that $F = 100$, $r = 4\%$, $Fr = 4$, $n = 30$ and $i = 4.5\%$. Let $k$ be the number of the half year when the bond is called.
If $k = 30$, then the price is

$$P_{30} = 4a_{\overline{30}|4.5\%} + 105(1.045)^{-30} = 93.19.$$



 

## Example 1

*Consider a 100 par–value 8% bond with semiannual coupons callable at 120 on any coupon date starting 5 years after issue for the next 5 years, at 110 starting 10 years after issue for the next 5 years and maturing at 105 at the end of 15 years. What is the highest price which an investor can pay and still be certain of a yield of 9% converted semiannually.*

**Solution:** We have that $F = 100$, $r = 4\%$, $Fr = 4$, $n = 30$ and $i = 4.5\%$. Let $k$ be the number of the half year when the bond is called.

We conclude that the highest price which an investor can pay and still be certain of a yield of 9% converted semiannually is 93.19.

## Example 2

*Joshua paid 800 for a 15-year 1000 par value bond with semiannual coupons at a nominal annual rate of 4% convertible semiannually. The bond can be called at 1300 on any coupon date starting at the end of year 7. What is the minimum annual nominal rate convertible semiannually yield that Joshua could receive?* **Answer:** 7.3521%.

## Example 2

*Joshua paid 800 for a 15-year 1000 par value bond with semiannual coupons at a nominal annual rate of 4% convertible semiannually. The bond can be called at 1300 on any coupon date starting at the end of year 7. What is the minimum annual nominal rate convertible semiannually yield that Joshua could receive?* **Answer:** 7.3521%.

**Solution:** We have that $F = 1000$, $P = 800$, $C = 1300$, $r = 0.02$ and $n = 30$. Since $P < C$, the bond was bought at discount. We assume that the redemption value is as late as possible. From the equation

$$800 = 20a_{\overline{30}|i} + 1300(1+i)^{-30}$$

we get that $i = 3.6760$ and $i^{(2)} = 7.3521\%$.



 