Manual for SOA Exam FM/CAS Exam 2.

Chapter 5. Bonds.

Section 5.6. More securities.

Extract from:
"Arcones' Manual for the SOA Exam FM/CAS Exam 2,
Financial Mathematics. Fall 2009 Edition",
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Serial bonds

A **serial bond** is a collection of bonds issued at the same time but with different redemption dates. The price of a serial bond is the sum of the prices of the individual bonds.
Let P_k, C_k, K_k be the price value, the redemption value and the present value of redemption value of the k–th bond. Let P', C', K' be the price value, redemption value and present value of redemption value of the serial bond. For each k we have that

$$P_k = K_k + \frac{g}{i}(C_k - K_k).$$

Let $P' = \sum_{j=1}^{k} P_j$, $C' = \sum_{j=1}^{k} C_j$ and $K' = \sum_{j=1}^{k} K_j$. Hence,

$$P' = K' + \frac{g}{i}(C' - K').$$

Example 1

A 12% serial bond with semiannual coupons and par value of 1000 will be redeemed by the following schedule:
(i) 100 at the end of years 10 through 14; and
(ii) 500 at the end of year 15.
Calculate the price of the bond on the issue date to yield 10% per annum convertible semiannually.

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Solution: We have that $g = \frac{Fr}{C} = 6\%$ and $i = 5\%$. Since the annual nominal rate compounded semiannually is 10%, the annual effective rate of interest is 10.25%. The redemption values and times of redemption are given by the following table:

Redemption value	100	100	100	100	100	500
Time in years	10	11	12	13	14	15

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Hence,

$$K' = \sum_{k=10}^{14} (100)(1+0.1025)^{-k} + (500)(1+0.1025)^{-15}$$
$$= (100)(1.1025)^{-9} a_{\overline{5}|10.25\%} + (500)(1.1025)^{-15}$$
$$= 156.51 + 115.69 = 272.20.$$

Hence,

$$P' = K' + \frac{g}{i}(C' - K') = 272.20 + \frac{0.06}{0.05}(1000 - 272.20) = 1145.56.$$

Preferred stock

Preferred stock is like a perpetual bond. This stock pays dividends forever. The price of the stock is the present value of future dividends. If a preferred stock pays an annual dividend D, then the price of this stock is

$$P = Da_{\overline{\infty}|i} = \frac{D}{i},$$

where i the annual effective rate of interest.

Example 2

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Solution: $\frac{3000}{0.06} = 50000$.

Common stock

For **common stock** the dividends are not known in advance. So one has to project what these dividends will be in the future. For example, let D be the dividend at the end of the current period and assume that the next dividends change geometrically with common ratio $1+k$ with $-1 < k < i$, then the cashflow of dividends is:

Contributions	D	$D(1+k)$	$D(1+k)^2$	$\cdots$
Time, in years	1	2	3	$\cdots$

The price of the common stock is

$$P = \frac{D}{1+k} a_{\overline{\infty}|\frac{i-k}{1+k}} = D\,\frac{1}{i-k}.$$

Recall that

$$P = D\frac{1}{1+i} + D\,\frac{1+k}{(1+i)^2} + D\,\frac{(1+k)^2}{(1+i)^3} + \ldots = \frac{D}{1+i} \cdot \frac{1}{1-\frac{1+k}{1+i}} = D\,\frac{1}{i-k}.$$

Example 3

Each quarter the corporation plans to pay 45% of its earnings as a stock dividend. The earnings of a corporation increase at 1% per quarter indefinitely. At the start of a quarter, an investor purchases the stock to yield a nominal rate of 5% compounded quarterly. The first stock dividend is 2.4 payable at the end of the quarter. Calculate the theoretical price of the stock.

Example 3

Each quarter the corporation plans to pay 45% of its earnings as a stock dividend. The earnings of a corporation increase at 1% per quarter indefinitely. At the start of a quarter, an investor purchases the stock to yield a nominal rate of 5% compounded quarterly. The first stock dividend is 2.4 payable at the end of the quarter. Calculate the theoretical price of the stock.

Solution: Since the earnings of a corporation increase at 1% per quarter, the dividends also increase at 1% per quarter. The cashflow of dividends is

Dividends	2.4	$(2.4)(1.01)$	$(2.4)(1.01)^2$	$\cdots$
Time	1	2	3	$\cdots$

The present value of the cashflow of dividends is $\frac{P}{(i^{(4)}/4)-k} = \frac{2.4}{(0.05/4)-0.01} = 960.$

