Manual for SOA Exam FM/CAS Exam 2.

Chapter 7. Derivatives markets.

Section 7.1. Derivatives.

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Extract from:
"Arcones' Manual for the SOA Exam FM/CAS Exam 2, Financial Mathematics. Fall 2009 Edition",
available at http://www.actexmadriver.com/

Risk sharing

- A **risk** is a contingent financial loss. Changes in commodity prices, currency exchange rates and interest rates are potential risks for a business. A farmer faces the possible fall of the price of his/her crop. Surging oil prices can wipe out airlines' profits. Manufacturing companies face high rising prices of commodities. These changes in prices could hurt the viability of a business.

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- Risk is **nondiversifiable** when it does vanish when spread across many investors.
- A way to do risk sharing for companies is to do contracts to avoid risks.

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Derivatives

Definition 1

A **derivative** *is a contract which specifies the right or obligation to receive or deliver certain asset for a certain price. The value of a derivative contract depends on the value of another asset.*

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- Regulatory arbitrage. Sometimes business enter into derivatives to get around regulatory limitations, accounting regulations and taxes.

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Example 1

Suppose that a farmer grows wheat and a baker makes bread using wheat and other ingredients. If the price of the wheat decreases, the farmer loses money. If the price of the wheat increases, the baker loses money. In order to avoid possible financial losses which may jeopardy their businesss profitability, the farmer and the baker can agree to sell/buy wheat one year from now at a certain price. The contract they enter is a derivative. It is a win–win contract for both of them. The two risks are diversifiable.

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Usually, the contract is not made directly between them. A **market–maker** *or* **scalper** *makes a contract with the farmer and another with the baker. The farmer and the baker enter into this contract to do* **hedging**.

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Derivatives in Practice.

- Derivatives are often traded on commodities, stock, stock indexes, currency exchange rates and interest rates. Common commodities in derivatives are agricultural (corn, soybeans, wheat, live cattle, cattle feeder, hogs lean, sugar, coffee, orange juice), metals (gold, silver, copper, lead, aluminum, platinum) and energy (crude oil, ethanol, natural gas, gasoline).

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- Derivative contracts for agricultural commodities have been traded in the U.S. for more than 100 years. The biggest markets in derivatives are the Chicago Board for Trade, the Chicago Mercantile Exchange, the New York Mercantile Exchange, and the Eurex (Frankfurt, Germany).

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- The market in derivatives is regulated by the (SEC) Securities and Exchange Commission and the (CFTC) Commodity Futures Trading Commission.

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Buying an asset

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- **bid–ask percentage spread** is the bid–ask spread divided over the ask price. Scalpers also ask for commissions.

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bid	price at which the scalper buys
ask or offer price	price at which the scalper sells

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Example 2

An online currency exchange service's bid rate for Japanese yens is \$0.00838 and its ask rate is \$0.00847. Find the bid–ask percentage spread.

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Solution: The ask percentage spread is $\frac{0.00847-0.00838}{0.00847} = 1.062574\%$.

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Example 3

$1 million face value six month T bill is traded by a government security dealer who give the following annual nominal discount yield convertible semiannually:

Bid	*Ask*
4.96%	*4.94%*

(i) Find the price the dealer is asking for the T bill
(ii) Find the price the dealer is buying the T bill.
(iii) Find the bid–ask percentage spread.

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Notice that the dealer sells the T bill for money than he buys it.

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(iii) The bid–ask percentage spread is
$\frac{975300-975200}{975300} = 0.01025325541\%$.

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Long and short positions

- When some one owns an asset, we say to this person has a **long position** in this asset. Later, he may sell the asset and receive cash. Buying an asset means to make an investment. Buying an asset is like lending money.

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Long and short positions

- When some one owns an asset, we say to this person has a **long position** in this asset. Later, he may sell the asset and receive cash. Buying an asset means to make an investment. Buying an asset is like lending money.
- When some one needs to buy an asset in the future, it is said that this person has a **short position** in the asset.

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Short sale

A **short sale** of an asset entails borrowing an asset and then immediately selling the asset receiving cash. Later, the short seller must buy back the asset paying cash for it and return it to the lender. The act of buying the replacement asset and return it to the lender is called to **close** or **cover** the short position.

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believe	derivative	desired outcome
price will increase	purchase	buy low now and sell high later
price will decrease	short sale	sell high now and buy low later

Table: An investor makes money buying low and selling high.

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- Financing. A short sale is a way to borrow money.
- Hedging. A short sale can be undertaken to offset the risk of owning an asset.

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- Scarcity of shares.

If the stock pays dividends, the short seller must return the paid dividend payments to the stock lender.

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Example 4

Mary short sells 200 shares of XYZ stock which has a bid price of $18.12 and ask price of $18.56. Her broker charges her a 0.5% commission to take on the short sale. Mary covers her position 6 months later when the bid price is $15.74 and the ask price is $15.93. The commission to close the short sale is $15. XYZ stock did not pay any dividends in those six months. How much does Mary earn in this short sale?

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Example 4

Mary short sells 200 shares of XYZ stock which has a bid price of $18.12 and ask price of $18.56. Her broker charges her a 0.5% commission to take on the short sale. Mary covers her position 6 months later when the bid price is $15.74 and the ask price is $15.93. The commission to close the short sale is $15. XYZ stock did not pay any dividends in those six months. How much does Mary earn in this short sale?

Solution: Mary short sells the stock for

$$(200)(18.12)(1-0.005) = \$3605.88.$$

Mary covers her position for

$$(200)(15.93)+15 = \$3201.$$

Mary earns $3605.88 - 3201 = \$404.88$.

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- Often, short sellers are required to make a deposit as collateral into an account with the lender. This account is called the **margin account**. This deposit is called the **margin requirement**. Usually, the margin requirement is a percentage of the current price of the stock. The margin requirement could be bigger than the current asset price. If this is so, the excess of the margin over the current asset price is called **haircut**.

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- This margin account generates an interest for the investor. Demand on short sales is a factor to determine the **margin account interest rate**. The **margin interest rate** is called the **repo rate** for bonds and the **short rebate** for stock.

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- This margin account generates an interest for the investor. Demand on short sales is a factor to determine the **margin account interest rate**. The **margin interest rate** is called the **repo rate** for bonds and the **short rebate** for stock.
- When borrowing, the lender can require payment of certain benefits lost by lending the asset. This payment requirement is called the **lease rate** of the asset. Usually, the lease rate of a stock is the payment of the dividends obtained while the stock was shorted. Usually, the lease rate of a bond is the payment of the coupons obtained while the bond was shorted.

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Sometimes short sales are subject to a margin requirement (or deposit). The investor has to set–up an account with a percentage of the current price of the stock. This margin account generates an interest for the investor. If the stock which the investor borrows pays a dividend, the investor must pay the dividend to the brokerage firm making the loan.
We have the following variables in a short sale of a stock:

- $P =$ profit on sale=price sold−price bought.
- $M =$ margin requirement= deposit on the short sale.
- $D =$ dividend paid by the short seller to the security's owner.
- $j =$ rate of interest earned in the margin account.
- $I = Mj$ interest earned by the short seller on the margin deposit.
- $i =$ yield rate on the short sale.

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We have that the net profit is

Net profit = gain on short sale+interest on margin−dividend on stock =

Hence, the yield rate earner in a short sale is

$$i = \frac{\text{net profit}}{\text{margin}} = \frac{P + I - D}{M}.$$

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Example 5

Jason sold short 1,000 shares of FinanTech at $75 a share on January 2, 2006. Jason is required to hold a margin account with his broker equal to 50% of the short security's initial value. Jason's margin account earns an annual effective interest rate of i. There is a $0.25 per share dividend paid on December 31, 2006. On January 2, 2007, Jason buys back stock to cover his position at a price of $70 per share. Jason's annual effective yield in this investment is 17.6667%. Find i.

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Solution: We have that $P = (75)(1000) - (70)(10000) = 5000$, $M = (75)(1000)(0.50) = 37500$, $I = 37500i$ and $D = (1000)(0.25) = 250$. Jason's annual effective yield is

$$0.176667 = \frac{P + I - D}{M} = \frac{5000 + 37500i - 250}{37500}.$$

Hence, $i = \frac{(0.176667)(37500) - 5000 + 250}{37500} = 5\%$.

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