

Manual for SOA Exam FM/CAS Exam 2.
Chapter 7. Derivative markets.
Section 7.3. Futures.

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Futures

A **future** is a standardized contract in which two counterparts agree to buy/sell an asset for a specified price (the future price) at a specified date (the delivery date).

The buyer in the future contract is called the **long future**. The seller in the future contract is called the **short future**.

The main reasons to enter into a future contract are hedging and speculation.

At difference of futures, forward contracts are privately negotiated and are not standardized. Forward contracts are entirely flexible. Forward contracts are tailor-made contracts.

Futures are bought and sold in organized futures exchanges. The biggest future exchanges are the Chicago Mercantile Exchange, the Chicago Board of Trade, the International Petroleum Exchange of London, the New York Mercantile Exchange, the London Metal Exchange and the Tokyo Commodity Exchange.

Futures transactions in the USA are regulated by the (CFTC) Commodity Futures Trading Commission, an agency of the USA government. The CFTC also regulates option markets.

A future contract is negotiated through a brokerage firm that holds a seat on the exchange. A future contract is settled by a clearinghouse owned by or associated with the exchange. The clearinghouse matches the purchases and the sales which take place during the day. By matching trades, the clearinghouse never takes market risk because it always has offsetting positions with different counterparts. By having the clearinghouse as counterpart, an individual entering a future contract does not face the possible credit risk of its counterpart.

Let us consider some common futures.

Crude oil futures trade in units of 1,000 U.S. barrels (42,000 gallons). The underlying is a US barrel. The notional amount is 1000 barrels. The current price is \$70/barrel. Hence, the current value of a future contract on crude oil is \$70000.

S & P 500 future contracts trade on 250 units of the index. They are cash settled. At expiration time, instead of a sale, one of the future counterpart receive a payment according with S & P 500 spot price at expiration. The current price of S & P 500 is 1500. The current value of a future contract on S & P 500 is $(250)(1500) = \$375000$.

Suppose that two parties agree in a future contract for crude oil for delivery in 18 months. The contract is worth \$70000. Each (investor) party makes a trade with the clearinghouse. This contract has two risks: **market risk** and **credit risk**. The market risk is related with the volatility of the price of the asset. The credit risk is related with the solvency of each party. To avoid credit risk, an individual or corporation entering a future contract must make a deposit into an account called the **margin account**. This deposit is called the **initial margin**. The margin account earns interest. The amount of the initial margin is determined by the exchange. It is usually a fraction of the market value of the futures' underlying asset. Usually future positions are settled into the margin account either every day or every week. By every day we mean every day which the market is open. Let us suppose that a clearinghouse settles accounts daily. Suppose that the annual continuously compounded interest rate is r .

Every day, the profit or loss is calculated on the investor's futures position. If there exists a loss, the investor's broker transfers that amount from the investor's margin account to the clearinghouse. If a profit, the clearinghouse transfers that amount to investor's broker who then deposits it into the investor's margin account. The profit for a long position in a future contract is

$$M_{t-(1/365)}(e^{r/365} - 1) + N(S_t - S_{t-(1/365)}),$$

where $M_{t-(1/365)}$ is the yesterday's balance in the margin account, N is the nominal amount, S_t is the current price, $S_{t-(1/365)}$ is the yesterday price. Hence, after the settlement, the balance in the investor's margin account is

$$M_t = M_{t-(1/365)}e^{r/365} + N(S_t - S_{t-(1/365)}).$$

The profit for a short position in a future contract is

$$M_{t-(1/365)}(1 - e^{r/365}) + N(S_{t-(1/365)} - S_t).$$

Marking-to-market is to calculate the value of a future contract according with the current value of the asset.

Example 1

On July 5, 2007, John enters a long future contract for 1,000 U.S. barrels of oil at \$71.6 a barrel. The margin account is 50% of the market value of the futures' underlier. The annual continuously compounded rate of return is 0.06.

- (i) On July 6, 2007, the price of oil is \$70.3. What is the balance in John's margin account after settlement?*
- (ii) On July 7, 2007, the price of oil is \$72.1. What is the balance in John's margin account after settlement?*

Solution: (i) The initial balance in John's margin account is

$$(0.50)(1000)(71.6) = 35800.$$

The balance in John's margin account on July 6, 2007, after settlement, is

$$\begin{aligned} & M_{t-(1/365)} e^{r/365} + N(S_t - S_{t-(1/365)}) \\ & = (35800) e^{0.06/365} + (1000)(70.3 - 71.6) = 35105.89. \end{aligned}$$

Since the price of the oil decreases, the value of having 1000 barrels in 18 months decreases.

Solution: (ii) The balance in John's margin account on July 6, 2007, after settlement, is

$$\begin{aligned} & M_{t-(1/365)} e^{r/365} + n(S_t - S_{t-(1/365)}) \\ & = (35105.89) e^{0.06/365} + (1000)(72.1 - 70.3) = 35711.56. \end{aligned}$$

Notice that this balance is different from

$$(35800) e^{(0.06)(2/365)} + (1000)(72.1 - 71.6) = 36311.77.$$

In the first day, John's account balance was smaller. So, John lost interest because the drop on price on July 6, 2007.

If the balance in the margin account falls the clearinghouse has less protection against default. Investors are required to keep the margin account to a minimum level. This level is a fraction of the initial margin. The **maintenance margin** is the fraction of the initial margin which participants are asked to hold in their accounts. If the balance in the margin account falls below this level, an investor's broker will require the investor to deposit funds sufficient to restore the balance to the initial margin level. Such a demand is called a **margin call**. If an investor fail to the deposit, the investor's broker will immediately liquidate some or all of the investor's positions.

Example 2

A company enters into a short futures contract to sell 100,000 pounds of frozen orange juice for \$1.4 cents per pound. The initial margin is 30% and the maintenance margin is 20%. The annual effective rate of interest is 4.5%. The account is settled every week. What is the minimum next week price which would lead to a margin call?

Solution: The initial balance in the margin account is $(0.30)(100000)(1.4) = 42000$. The minimum balance in the margin account is $(0.20)(100000)(1.4) = 28000$. After settlement next week balance is

$$42000(1.045)^{1/52} + 100000(1.4 - S_{1/52}).$$

A margin call happens if

$$28000 > 42000(1.045)^{1/52} + 100000(1.4 - S_{1/52}),$$

or

$$S_{1/52} > 1.4 - \frac{28000 - 42000(1.045)^{1/52}}{100000} = 1.540355672.$$

Besides holding the contract until expiration, there are two ways to close a future contract: offset the contract and exchange for physicals. To offset the contract means to enter a reverse position with the same broker. Since future contracts are standardized, it is possible to find a reverse position on a contract. Exchange for physicals consists selling/buying the commodity.

The two main advantages of futures versus forwards are liquidity and counter-party risk. It is much easier to cancel before expiration a future contract than a forward contract. Since the trade is made against a clearinghouse, a participant does face credit risk. At the same time, the margin and the marking to market reduces the default risk.

Having a margin account makes the profit/losses of the investment higher for a future than for a forward. The oscillations of the price of the asset make the earnings in the margin account more variable. Usually, if there is a profit from the change of the price of the asset, there is also a profit in the balance account. Reciprocally, if there is a loss from the price change, there exists a loss in the margin account.