Manual for SOA Exam FM/CAS Exam 2.

Chapter 7. Derivatives markets.

Section 7.6. Put–call parity.

Extract from:
"Arcones' Manual for the SOA Exam FM/CAS Exam 2, Financial Mathematics. Fall 2009 Edition",
available at http://www.actexmadriver.com/

Put–call parity

Recall that the actions and payoffs corresponding to a call/put are:

	If $S_T < K$	If $K < S_T$
long call	no action	buy the stock
short call	no action	sell the stock
long put	sell the stock	no action
short put	buy the stock	no action

	If $S_T < K$	If $K < S_T$
long call	0	$S_T - K$
short call	0	$-(S_T - K)$
long put	$K - S_T$	0
short put	$-(K - S_T)$	0

- If we have a K–strike long call and a K–strike short put, we are able to buy the asset at time T for K. Hence, **having both a K–strike long call and a K–strike short put is equivalent to have a K–strike long forward contract with price K.**
- Entering into both a K–strike long call and a K–strike short put is called a **synthetic long forward.**
- Reciprocally, if we have a K–strike short call and a K–strike long put, we are able to sell the asset at time T for K. **Having both a K–strike short call and a K–strike long put is equivalent to have a short forward contract with price K.**
- Entering into both a K–strike short call and a K–strike long put is called a **synthetic short forward.**

The no arbitrage cost at time T of buying an asset using a long forward contract is $F_{0,T}$. The cost at time T for buying an asset using a K–strike long call and a K–strike short put is

$$(\mathrm{Call}(K, T) - \mathrm{Put}(K, T))e^{rT} + K.$$

If there exists no arbitrage, then:

Theorem 1

(Put–call parity formula)

$$(\mathrm{Call}(K, T) - \mathrm{Put}(K, T))e^{rT} + K = F_{0,T}.$$

If we use effective interest, the put–call parity formula becomes:

$$(\mathrm{Call}(K, T) - \mathrm{Put}(K, T))(1 + i)^{T} + K = F_{0,T}.$$

Often, $F_{0,T} = S_0(1+i)^T$. This forward price applies to assets which have neither cost nor benefit associated with owning them. In the absence of arbitrage, we have the following relation between call and put prices:

Theorem 2

*(**Put–call parity formula**) For a stock which does not pay any dividends,*

$$(\mathrm{Call}(K,T) - \mathrm{Put}(K,T))e^{rT} + K = S_0 e^{rT}.$$

Proof.

Consider the portfolio consisting of buying one share of stock and a K–strike put for one share; selling a K–strike call for one share; and borrowing $S_0 - \mathrm{Call}(K, T) + \mathrm{Put}(K, T)$. At time T, we have the following possibilities:

1. If $S_T < K$, then the put is exercised and the call is not. We finish without stock and with a payoff for the put of K.
2. If $S_T > K$, then the call is exercised and the put is not. We finish without stock and with a payoff for the call of K.

In any case, the payoff of this portfolio is K. Hence, K should be equal to the return in an investment of
$S_0 + \mathrm{Put}(K, T) - \mathrm{Call}(K, T)$ in a zero–coupon bond, i.e.
$K = (S_0 + \mathrm{Put}(K, T) - \mathrm{Call}(K, T))e^{rT}$. □

Example 1

The current value of XYZ stock is 75.38 *per share. XYZ stock does not pay any dividends. The premium of a nine–month 80–strike call is 5.737192 per share. The premium of a nine–month 80–strike put is 7.482695 per share. Find the annual effective rate of interest.*

Example 1

The current value of XYZ stock is 75.38 *per share. XYZ stock does not pay any dividends. The premium of a nine–month 80–strike call is 5.737192 per share. The premium of a nine–month 80–strike put is 7.482695 per share. Find the annual effective rate of interest.*

Solution: The put–call parity formula states that

$$(\mathrm{Call}(K,T) - \mathrm{Put}(K,T))(1+i)^T + K = S_0(1+i)^T.$$

So,

$$(5.737192 - 7.482695)(1+i)^{3/4} + 80 = 75.38(1+i)^T.$$

$80 = (75.38 - (5.737192 - 7.482695))(1+i)^{3/4} =$
$(77.125503)(1+i)^{3/4}$, and $i = 5\%$.

Example 2

The current value of XYZ stock is 85 *per share. XYZ stock does not pay any dividends. The premium of a six–month K–strike call is 3.329264 per share and the premium of a one year K–strike put is 10.384565 per share. The annual effective rate of interest is* 6.5%. *Find K.*

Example 2

The current value of XYZ stock is 85 *per share. XYZ stock does not pay any dividends. The premium of a six–month K–strike call is 3.329264 per share and the premium of a one year K–strike put is 10.384565 per share. The annual effective rate of interest is* 6.5%. *Find K.*

Solution: The put–call parity formula states that

$$(\mathrm{Call}(K, T) - \mathrm{Put}(K, T))(1 + i)^T + K = S_0(1 + i)^T.$$

So, $(3.329264 - 10.384565)(1.065)^{0.5} + K = 85(1.065)^{0.5}$ and

$$K = (85 - 3.329264 + 10.384565)(1.065)^{0.5} = 95.$$

Example 3

XYZ stock does not pay any dividends. The price of a one year forward for one share of XYZ stock is 47.475. The premium of a one year 55*–strike put option of XYZ stock is 9.204838 per share. The annual effective rate of interest is* 5.5%*. Calculate the price of a one year* 55*–strike call option for one share of XYZ stock.*

Example 3

XYZ stock does not pay any dividends. The price of a one year forward for one share of XYZ stock is 47.475. The premium of a one year 55*–strike put option of XYZ stock is 9.204838 per share. The annual effective rate of interest is* 5.5%*. Calculate the price of a one year* 55*–strike call option for one share of XYZ stock.*

Solution: The put–call parity formula states that

$$(\mathrm{Call}(K, T) - \mathrm{Put}(K, T))(1 + i)^T + K = F_{0,T}.$$

So, $(\mathrm{Call}(55, 1) - 9.204838)(1.055) + 55 = 47.475$ and

$$\mathrm{Call}(55, 1) = 9.204838 + (47.475 - 55)(1.055)^{-1} = 2.072136578.$$

If prices of put options and call options do not satisfy the put–call parity, it is possible to do arbitrage.

- If
$$(S_0 - \mathrm{Call}(K,T) + \mathrm{Put}(K,T))e^{rT} > K,$$
we can make a profit by buying a call option, selling a put option and shorting stock. The profit of this strategy is
$$= -K + (S_0 - \mathrm{Call}(K,T) + \mathrm{Put}(K,T))e^{rT}.$$
- If
$$(S_0 - \mathrm{Call}(K,T) + \mathrm{Put}(K,T))e^{rT} < K,$$
we can do arbitrage by selling a call option, buying a put option and buying stock. At expiration time, we get rid of the stock by satisfying the options and make
$$K - (S_0 - \mathrm{Call}(K,T) + \mathrm{Put}(K,T))e^{rT}.$$

Example 4

XYZ stock trades at \$54 per share. XYZ stock does not pay any dividends. The cost of an European call option with strike price \$50 and expiration date in three months is \$8 per share. The risk free annual interest rate continuously compounded is 4%.

Example 4

XYZ stock trades at \$54 per share. XYZ stock does not pay any dividends. The cost of an European call option with strike price \$50 and expiration date in three months is \$8 per share. The risk free annual interest rate continuously compounded is 4%.

(i) Find the no–arbitrage price of a European put option with the same strike price and expiration time.

Example 4

XYZ stock trades at \$54 per share. XYZ stock does not pay any dividends. The cost of an European call option with strike price \$50 and expiration date in three months is \$8 per share. The risk free annual interest rate continuously compounded is 4%.

(i) Find the no–arbitrage price of a European put option with the same strike price and expiration time.

Solution: (i) With continuous interest, the put–call parity formula is

$$(\mathrm{Call}(K, T) - \mathrm{Put}(K, T))e^{rT} + K = S_0 e^{rT}.$$

Hence,

$$(8 - \mathrm{Put}(50, 0.25))e^{0.04(0.25)} + 50 = 54e^{0.04(0.25)}$$

and $\mathrm{Put}(50, 0.25) = 8 - 54 + 50e^{-0.04(0.25)} = 3.502491687$.

Example 4

XYZ stock trades at \$54 per share. XYZ stock does not pay any dividends. The cost of an European call option with strike price \$50 and expiration date in three months is \$8 per share. The risk free annual interest rate continuously compounded is 4%.

(ii) Suppose that the price of an European put option with the same strike price and expiration time is \$3, find an arbitrage strategy and its profit per share.

Example 4

XYZ stock trades at \$54 per share. XYZ stock does not pay any dividends. The cost of an European call option with strike price \$50 and expiration date in three months is \$8 per share. The risk free annual interest rate continuously compounded is 4%.

(ii) Suppose that the price of an European put option with the same strike price and expiration time is \$3, find an arbitrage strategy and its profit per share.

Solution: (ii) A put option for \$3 per share is undervalued. An arbitrage portfolio consists in selling a call option, buying a put option and stock and borrowing $\$-8+3+54=\$49$, with all derivatives for one share of stock. At redemption time, we sell the stock and use it to execute the option which will be executed. We also repaid the loan. The profit is $50-(49)e^{(0.04)(0.25)}=50-49.49245819=0.50754181$.

Example 5

Suppose that the current price of XYZ stock is 31*. XYZ stock does not give any dividends. The risk free annual effective interest rate is* 10%*. The price of a three–month* 30*–strike European call option is \$*3*. The price of a three–month* 30*–strike European put option is \$*2.25*. Find an arbitrage opportunity and its profit per share.*

Example 5

Suppose that the current price of XYZ stock is 31*. XYZ stock does not give any dividends. The risk free annual effective interest rate is* 10%*. The price of a three–month* 30*–strike European call option is* \$3*. The price of a three–month* 30*–strike European put option is* \$2.25*. Find an arbitrage opportunity and its profit per share.*

Solution: We have that

$$\begin{aligned}&(S_0 + \mathrm{Put}(K,T) - \mathrm{Call}(K,T))(1+i)^T\\ =&(31 + 2.25 - 3)(1.1)^{0.25} = 30.97943909 > 30.\end{aligned}$$

We conclude that the put is overpriced relatively to the call. We can sell a put, buy a call and short stock. The profit per share is

$$(2.25 - 3 + 31)(1.1)^{0.25} - 30 = 0.9794390948.$$

Notice that at expiration time one of the options is executed and we get back the stock which we sold.

Synthetic forward.

Definition 1

A **synthetic long forward** *is the combination of buying a call and selling a put, both with the same strike price, amount of the asset and expiration date.*

The payments to get a synthetic long forward are

- $(\mathrm{Call}(K, T) - \mathrm{Put}(K, T))$ paid at time zero.
- K paid at time T.

The future value of these payments at time T is

$$K + (\mathrm{Call}(K, T) - \mathrm{Put}(K, T))(1 + i)^T.$$

The payment of long forward is $F_{0,T}$ paid at time T.
In the absence of arbitrage (put–call parity)

$$F_{0,T} = K + (\mathrm{Call}(K, T) - \mathrm{Put}(K, T))(1 + i)^T.$$

The premium of a synthetic long forward, i.e. the cost of entering this position, is

$$(\mathrm{Call}(K, T) - \mathrm{Put}(K, T)).$$

- If $F_{0,T} = K$, the premium of a synthetic long forward is zero. You will buying the asset at the estimated future value of the asset.
- If $F_{0,T} > K$, the premium of a synthetic long forward is positive. You will buying the asset lower than the estimated future value of the asset.
- If $F_{0,T} < K$, the premium of a synthetic long forward is negative. You will buying the asset higher than the estimated future value of the asset.

Constructive sale.

An investor owns stock. He would like to sell his stock. But, he does not want to report capital gains to the IRS this year. So, instead of selling this stock, he holds the stock, buys a K–strike put, sells a K–strike call, and borrows $K(1+i)^{-T}$. The payoff which he gets at time T is

$$\begin{aligned}&S_T + \max(K - S_T, 0) - \max(S_T - K, 0) - K\\ =&\max(S_T, K) - \max(S_T, K) = 0.\end{aligned}$$

At time zero, the investor gets

$$\mathrm{Call}(K, T) - \mathrm{Put}(K, T) + K(1+i)^{-T} = F_{0,T}(1+i)^{-T}.$$

At expiration time, the investor can use the stock to meet the option which will be executed. Practically, the investor sold his stock at time zero for $F_{0,T}(1+i)^{-T}$. According with current USA tax laws, this is considered a **constructive sale**. He will have to declare capital gains when the options are bought.

Floor.

Suppose that you own some asset. If the asset losses value in the future, you lose money. A way to insure this long position is to buy a put position. The purchase of a put option is called a **floor**. A floor guarantees a minimum sale price of the value of an asset.

- The profit of buying an asset is $S_T - S_0(1+i)^T$, which is the same as the profit of a long forward. The minimum profit of buying an asset is $-S_0(1+i)^T$.
- The profit for buying an asset and a put option is

$$S_T - S_0(1+i)^T + \max(K - S_T, 0) - \mathrm{Put}(K, T)(1+i)^T$$
$$= \max(S_T, K) - (S_0 + \mathrm{Put}(K, T))(1+i)^T.$$

The minimum profit for buying an asset and a put option is

$$K - (S_0 + \mathrm{Put}(K, T))(1+i)^T.$$

We know that $-S_0(1+i)^T < K - (S_0 + \mathrm{Put}(K, T))(1+i)^T$.

Here is the graph of the profit for buying an asset and a put option:

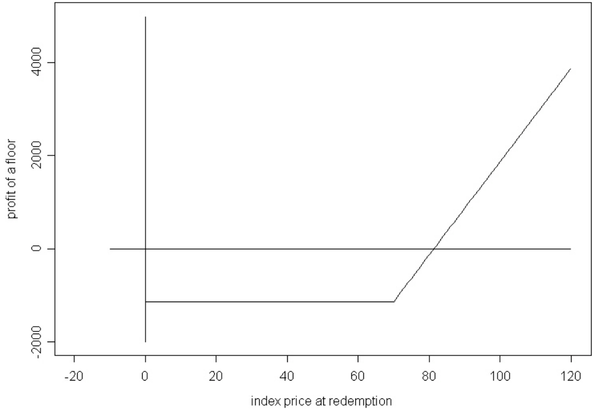

Figure 1: Profit for a long position and a long put.

Notice that this is the graph of the profit of a purchased call.

Here is a joint graph for the profits of the strategies: (i) buying an asset, (ii) buying an asset and a put.

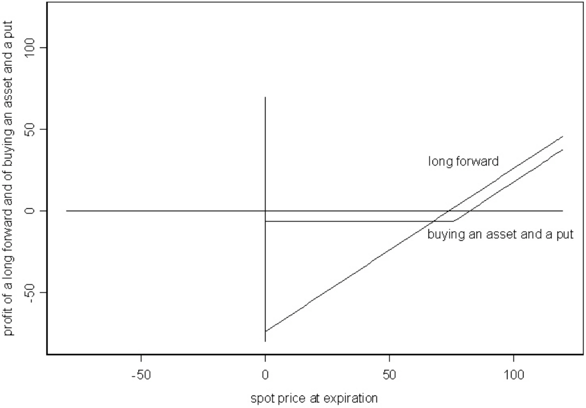

Figure 2: Profit for long forward and buying an asset and a put.

Table 1 was found using the Black–Scholes formula with $S_0 = 75$, $t = 1$, $\sigma = 0.20$, $\delta = 0$, $r = \log(1.05)$.

Table 1: Prices of some calls and some puts.

K	65	70	75	80	85
$\mathrm{Call}(K, T)$	14.31722	10.75552	7.78971	5.444947	3.680736
$\mathrm{Put}(K, T)$	1.221977	2.422184	4.218281	6.635423	9.633117

Example 6

The current price of one share of XYZ stock is \$75. Use the Table 1 for prices of puts and calls. Steve buys 500 shares of XYZ stock. Nicole buys 500 shares of XYZ stock and a put option on 500 shares of XYZ stock with a strike price 65 and an expiration date one year from now. The premium of a 65*–strike put option is* \$1.221977*. The risk free annual effective rate of interest is* 5%*.*

Example 6

The current price of one share of XYZ stock is \$75. Use the Table 1 for prices of puts and calls. Steve buys 500 shares of XYZ stock. Nicole buys 500 shares of XYZ stock and a put option on 500 shares of XYZ stock with a strike price 65 and an expiration date one year from now. The premium of a 65*–strike put option is* \$1.221977*. The risk free annual effective rate of interest is* 5%*.*

(i) Calculate the profits for Steve and Nicole.

Example 6

The current price of one share of XYZ stock is \$75. Use the Table 1 for prices of puts and calls. Steve buys 500 shares of XYZ stock. Nicole buys 500 shares of XYZ stock and a put option on 500 shares of XYZ stock with a strike price 65 and an expiration date one year from now. The premium of a 65*–strike put option is* \$1.221977*. The risk free annual effective rate of interest is* 5%*.*
(i) Calculate the profits for Steve and Nicole.

Solution: (i) Steve's profit is $(500)(S_T - 75)$. Nicole's profit is

$$\begin{aligned} &(500)(S_T - 75 + \max(65 - S_T, 0) - 1.221977(1.05)) \\ =&(500)(\max(65, S_T) - 76.28307585). \end{aligned}$$

Example 6

The current price of one share of XYZ stock is \$75. Use the Table 1 for prices of puts and calls. Steve buys 500 shares of XYZ stock. Nicole buys 500 shares of XYZ stock and a put option on 500 shares of XYZ stock with a strike price 65 and an expiration date one year from now. The premium of a 65*–strike put option is* \$1.221977*. The risk free annual effective rate of interest is* 5%*.*
(ii) Find a table with Steve's and Nicole's profits for the following spot prices at expiration: 60, 65, 70, 75, 80 and 85.

Example 6

The current price of one share of XYZ stock is \$75. Use the Table 1 for prices of puts and calls. Steve buys 500 shares of XYZ stock. Nicole buys 500 shares of XYZ stock and a put option on 500 shares of XYZ stock with a strike price 65 and an expiration date one year from now. The premium of a 65*–strike put option is* \$1.221977*. The risk free annual effective rate of interest is* 5%*.*
(ii) Find a table with Steve's and Nicole's profits for the following spot prices at expiration: 60, 65, 70, 75, 80 and 85.
Solution: (ii)

Spot Price	60	65	70	75	80	85
Steve's prof	−7500	−5000	−2500	0	2500	5000
Nicole's prof	−5641.54	−5641.54	−3141.54	−641.54	1858.46	4358.46

Example 6

The current price of one share of XYZ stock is \$75. Use the Table 1 for prices of puts and calls. Steve buys 500 shares of XYZ stock. Nicole buys 500 shares of XYZ stock and a put option on 500 shares of XYZ stock with a strike price 65 and an expiration date one year from now. The premium of a 65*–strike put option is* \$1.221977*. The risk free annual effective rate of interest is* 5%*.*
(iii) Calculate the minimum and maximum profits for Steve and Nicole.

Example 6

The current price of one share of XYZ stock is \$75. Use the Table 1 for prices of puts and calls. Steve buys 500 shares of XYZ stock. Nicole buys 500 shares of XYZ stock and a put option on 500 shares of XYZ stock with a strike price 65 and an expiration date one year from now. The premium of a 65*–strike put option is* \$1.221977*. The risk free annual effective rate of interest is* 5%*.*
(iii) Calculate the minimum and maximum profits for Steve and Nicole.

Solution: (iii) The minimum and maximum of Steve's profit are $(500)(-75) = -37500$ and ∞, respectively. The minimum Nicole's profit is $(500)(65 - 76.28307585) = -5641.537925$. The maximum Nicole's profit is ∞.

Example 6

The current price of one share of XYZ stock is \$75. Use the Table 1 for prices of puts and calls. Steve buys 500 shares of XYZ stock. Nicole buys 500 shares of XYZ stock and a put option on 500 shares of XYZ stock with a strike price 65 and an expiration date one year from now. The premium of a 65*–strike put option is* \$1.221977*. The risk free annual effective rate of interest is* 5%*.*
(iv) Find spot prices at expiration at which each of Steve makes a profit. Answer the previous question for Nicole.

Example 6

The current price of one share of XYZ stock is \$75. Use the Table 1 for prices of puts and calls. Steve buys 500 shares of XYZ stock. Nicole buys 500 shares of XYZ stock and a put option on 500 shares of XYZ stock with a strike price 65 and an expiration date one year from now. The premium of a 65*–strike put option is* \$1.221977*. The risk free annual effective rate of interest is* 5%*.*
(iv) Find spot prices at expiration at which each of Steve makes a profit. Answer the previous question for Nicole.

Solution: (iv) Steve's profit is positive if $(500)(S_T - 75) > 0$, i.e. if $S_T > 75$. Nicole's profit is positive if $(500)(\max(65, S_T) - 76.28307585) > 0$, ie. if $S_T > 76.28307585$.

Next, we proof that the profit of buying an asset and a put is the same as the profit of buying a call option. The payoff for buying a call option and a zero–coupon bond which pays the strike price at expiration date is

$$\max(0, S_T - K) + K = \max(S_T, K)$$

The payoff for buying stock and a put option is

$$S_T + \max(K - S_T, 0) = \max(S_T, K).$$

Since the two strategies have the same payoff in the absence of arbitrage, they have the same profit, i.e.

$$\begin{aligned} &\max(S_T, K) - K - (\mathrm{Call}(K, T))(1+i)^T \\ =&\max(S_T, K) - (S_0 + \mathrm{Put}(K, T))(1+i)^T. \end{aligned}$$

This equation is equivalent to the put–call parity:

$$(S_0 + \mathrm{Put}(K, T))(1+i)^T = K + \mathrm{Call}(K, T)(1+i)^T.$$

Example 7

Michael buys 500 shares of XYZ stock and a 45–strike four–year put for 500 shares of XYZ stock. Rita buys a 45–strike four–year call for 500 shares of XYZ stock and invests P into a zero–coupon bond. The annual rate of interest continuously compounded is 4.5%. Find P so that Michael and Rita have the same payoff at expiration.

Example 7

Michael buys 500 shares of XYZ stock and a 45–strike four–year put for 500 shares of XYZ stock. Rita buys a 45–strike four–year call for 500 shares of XYZ stock and invests P into a zero–coupon bond. The annual rate of interest continuously compounded is 4.5%. Find P so that Michael and Rita have the same payoff at expiration.

Solution: Michael's payoff at expiration is

$$500(S_4 + \max(45 - S_4, 0)) = 500 \max(45, S_4).$$

Rita's payoff at expiration is

$$\begin{aligned} &500 \max(S_4 - 45, 0) + Pe^{(0.045)(4)} \\ =&500 \max(45, S_4) - (500)(45) + Pe^{(0.045)(4)}. \end{aligned}$$

Hence, $P = (500)(45)e^{-(0.045)(4)} = 18793.57976.$

Cap.

If you have an obligation to buy stock in the future, you have a short position on the stock. You will experience a loss, when the price of the stock price rises. You can insure a short position by purchasing a call option. Buying a call option when you are in a short position is called a **cap**. The payoff of having a short position and buying a call option is

$$-S_T + \max(S_T - K, 0) = \max(-S_T, -K) = -\min(S_T, K).$$

The payoff of having a purchased put combined with borrowing the strike price at closing is

$$\max(K - S_T, 0) - K = \max(-S_T, -K) = -\min(S_T, K),$$

which is the same as before.

Suppose that you have a short position on a stock. Consider the following two strategies:

1. Buy a call option. The payoff at expiration is

$$-S_T + \max(S_T - K, 0) = \max(-S_T, -K).$$

The profit at expiration is

$$\max(-S_T, -K) - \mathrm{Call}(K, T)(1 + i)^T.$$

2. Buy stock (to cover the short) and a put option. The payoff at expiration is

$$\max(K - S_T, 0) = K + \max(-S_T, -K).$$

The profit at expiration is

$$K + \max(-S_T, -K) - (S_0 + \mathrm{Put}(K, T))(1 + i)^T.$$

By the put–call parity formula, both strategies have the same profit.

Example 8

Heather has a short position in 1200 shares of stock XYZ. She is supposed to return this stock one year from now. To insure her position, she buys a \$65–strike call. The premium of a \$65–strike call is \$14.31722. The current price of one share of XYZ stock is \$75. The risk free annual effective rate of interest is 5%.

Example 8

Heather has a short position in 1200 shares of stock XYZ. She is supposed to return this stock one year from now. To insure her position, she buys a \$65–strike call. The premium of a \$65–strike call is \$14.31722. The current price of one share of XYZ stock is \$75. The risk free annual effective rate of interest is 5%.

(i) Make a table with Heather's profit when the spot price at expiration is \$40, \$50, \$60, \$70, \$80, \$90.

Example 8

Heather has a short position in 1200 shares of stock XYZ. She is supposed to return this stock one year from now. To insure her position, she buys a \$65–strike call. The premium of a \$65–strike call is \$14.31722. The current price of one share of XYZ stock is \$75. The risk free annual effective rate of interest is 5%.

(i) Make a table with Heather's profit when the spot price at expiration is \$40, \$50, \$60, \$70, \$80, \$90.

Solution: (i) Heather's profit is

$$
\begin{aligned}
&(1200)\left(\max(-S_T, -K) - \mathrm{Call}(K, T)(1+i)^T\right)\\
=&1200(\max(-S_T, -65) - 14.31722(1.05)^1)\\
=&1200(\max(-S_T, -65) - 15.033081200).
\end{aligned}
$$

Spot Price	40	50	60	70	80	90
H's profit	−66039.70	−78039.70	−90039.70	−96039.70	−96039.70	−96039.70

Example 8

Heather has a short position in 1200 shares of stock XYZ. She is supposed to return this stock one year from now. To insure her position, she buys a \$65–strike call. The premium of a \$65–strike call is \$14.31722. The current price of one share of XYZ stock is \$75. The risk free annual effective rate of interest is 5%*.*
(ii) Assuming that she does not buy the call, make a table with her profit when the spot price at expiration is \$40, \$50, \$60, \$70, \$80, \$90.

Example 8

Heather has a short position in 1200 shares of stock XYZ. She is supposed to return this stock one year from now. To insure her position, she buys a \$65–strike call. The premium of a \$65–strike call is \$14.31722. The current price of one share of XYZ stock is \$75. The risk free annual effective rate of interest is 5%*.*
(ii) Assuming that she does not buy the call, make a table with her profit when the spot price at expiration is \$40, \$50, \$60, \$70, \$80, \$90.
Solution: (ii) Heather's profit is $-(1200)S_T$.

Spot Price	40	50	60	70	80	90
H's profit	−48000	−60000	−72000	−84000	−96000	−108000

Example 8

Heather has a short position in 1200 shares of stock XYZ. She is supposed to return this stock one year from now. To insure her position, she buys a \$65–strike call. The premium of a \$65–strike call is \$14.31722. The current price of one share of XYZ stock is \$75. The risk free annual effective rate of interest is 5%*.*

(iii) Draw the graphs of the profit versus the spot price at expiration for the strategies in (i) and in (ii).

Example 8

Heather has a short position in 1200 shares of stock XYZ. She is supposed to return this stock one year from now. To insure her position, she buys a \$65–strike call. The premium of a \$65–strike call is \$14.31722. The current price of one share of XYZ stock is \$75. The risk free annual effective rate of interest is 5%.
(iii) Draw the graphs of the profit versus the spot price at expiration for the strategies in (i) and in (ii).
Solution: (iii) The graph of profits is on Figure 3.

Notice that the possible loss for the short position can be arbitrarily large. By buying the call Heather has limited her possible losses.

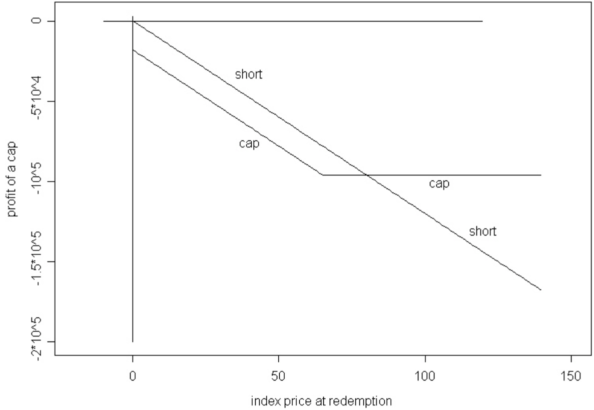

Figure 3: Profit for a long position and a long put.

Selling calls and puts.

Selling a option when there is a corresponding long position in the underlying asset is called **covered writing** or **option overwriting**. **Naked writing** occurs when the writer of an option does not have a position in the asset.

- A **covered call** is achieved by writing a call against a long position on the stock. An investor holding a long position in an asset may write a call to generate some income from the asset.
- A **covered put** is achieved by writing a put against a short position on the stock.

An arbitrageur buys and sells call and puts. A way to limit risks in the sales of options is to cover these positions. He also can match opposite options. For example,

- A purchase of K–strike call option, a sale of a K–strike put option and a short forward cancel each other.
- A sale of K–strike call option, a purchase of a K–strike put option and a long forward cancel each other.

