Manual for SOA Exam FM/CAS Exam 2.

Chapter 7. Derivatives markets.

7.7. Equity linked certificates of deposit.

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Equity linked certificates of deposit

To estimate the evolution of the stock market, stock indexes are used. A stock market index is a listing of stocks and a way to obtain the composite value of its components. The three most used stock indexes are the (Dow Jones) **Dow Jones Industrial Average**, (S & P 500) **Standard & Poor's 500 index**, and the **NASDAQ Composite Index**. Usually the composite value of an index is a sort of average of the stocks in the index. However, there are different ways to find this average. Every stock market index has its rules to find its composite value.

The S & P 500 index consisting of 500 stocks of large corporations selected by Standard & Poor. Standard & Poor is a financial company which specializes in providing independent credit rating and index evaluation. The stocks on the S & P 500 trade in stock markets, like the (NYSE) NYSE **New York Stock Exchange** and (National Association of Securities Dealers Automated Quotations system) NASDAQ. The New York Stock Exchange is the largest equities marketplace in the world. NASDAQ is an electronic stock exchange.

The Dow Jones Industrial Average is obtained "averaging" the value of 30 stocks selected by the Dow Jones & Company. These 30 stocks are selected from largest and the most widely held public companies in the USA across a range of industries except for transport and utilities. Dow Jones & Company publishes the The Wall Street Journal. The Wall Street Journal editors have a lot input on the selection of the stocks in the Dow Jones Industrial Average.

The NASDAQ Composite Index consists of all securities listed on NASDAQ. It contains mainly stocks of technology and growth companies.

Roughly, the difference between the three indexes is on the type of stocks which they represent. The S & P 500 focuses on all large–cap stocks in the market. The Dow Jones Industrial Average focuses on a very selected group of large companies. NASDAQ focuses on technology and fast growing companies.

An (ELCD) **equity linked CD** (or **equity linked note**, or **equity indexed CD**, or **market index linked CD**) is an FDIC–insured certificate of deposit that ties the rate of return to the performance of a stock index such as the S & P 500 and guarantee a certain payment. Chase Manhattan Bank first introduced ELCD's in 1987. But, now many financial institutions offer ELCD's. Usually, the guarantee payment is the original principal. The investor at expiration also gets a payment depending on the performance of the stock index. Usually, there exists a participation rate r, $0 < r \leq 1$, such that the investor gets the guarantee payment plus $rP \max\left(\frac{S_T}{S_0} - 1, 0\right)$, where P is the principal invested, S_T is the index price at expiration, S_0 is the spot price. Hence, usually, the payoff of an ELCD is

$$P\left(1 + r \max\left(\frac{S_T}{S_0} - 1, 0\right)\right).$$

An investor is attracted to ELCD's because it has the potential for market appreciation and diversification without risking capital. Diversification is attained by using market indices, which combine several stocks. Usually the return of the capital is FDIC insured. One disadvantage is the possible loss of interest on the invested principal. Notice that the smallest payoff which the investor may get is his invested principal, i.e. he does not get any interest. The instrument is appropriate for conservative equity investors or fixed income investors who desire equity exposure with controlled risk.

Example 1

Aaron deposits $15000 in an ELCD, which provides 100% principal protection and pays 80% of the appreciation of the S & P 500 three years from now. The index closes at 1500 on the day the ELCD is issued.

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Solution: (i) Aaron's payoff is

$$P\left(1+r\max\left(\frac{S_T}{S_0}-1,0\right)\right)$$

$$=(15000)\left(1+(0.80)\max\left(\frac{S_T}{1500}-1,0\right)\right).$$

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(ii) Find the Aaron's payoff in the forward contract if S_3 is \$1300, \$1400, \$1500, \$1600, \$1700, \$1800.

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Solution: (ii) Using that Aaron's payoff is

$$(15000)\left(1+(0.80)\max\left(\frac{S_T}{1500}-1,0\right)\right),$$

Payoff	15000	15000	15000	15800	16600	17400
S_3	1300	1400	1500	1600	1700	1800

Example 1

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(iii) Graph Aaron's payoff as a function of S_3.

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(iii) Graph Aaron's payoff as a function of S_3.

Solution: (iii) Aaron's payoff is Figure 1.

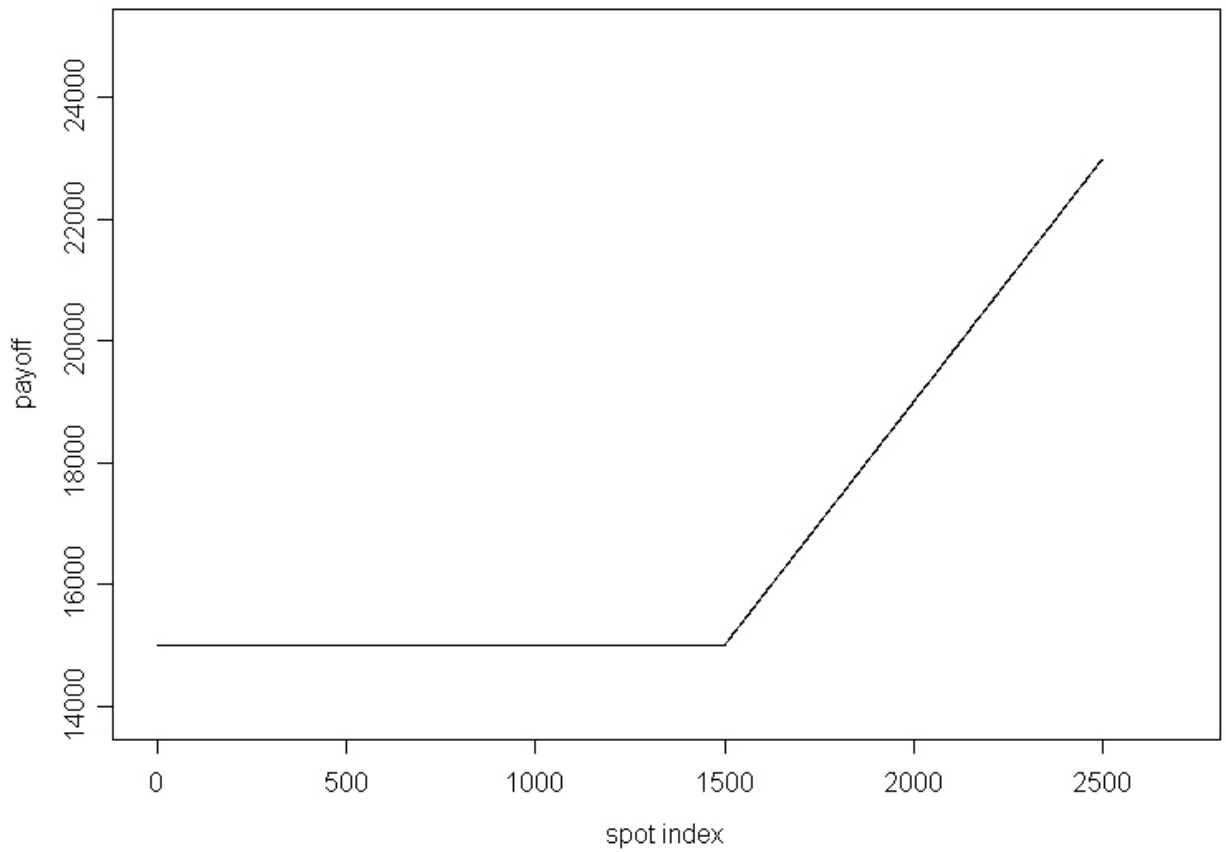

Figure 1: Example 1. ELCD payoff.

The payoff of an ELCD is a combination of the payoff of a long call option and a long bond position. It is possible to create a synthetic ELCD by buying a long call option and a zero–coupon bond.

The return on a ELCD is

$$P\left(1+r\max\left(\frac{S_T}{S_0}-1,0\right)\right)=P+\frac{Pr}{S_0}\max(S_T-S_0,0),$$

Suppose that an investor buys a zero–coupon bond with face value P and a S_0–strike call option for $\frac{Pr}{S_0}$ shares. His payoff at expiration date is

$$P+\frac{Pr}{S_0}\max(S_T-S_0,0).$$

The cost of making this investment is

$$P(1+i)^{-T}+\frac{Pr}{S_0}\mathrm{Call}(S_0,T).$$

If there exist no arbitrage,

$$P=P(1+i)^{-T}+\frac{Pr}{S_0}\mathrm{Call}(S_0,T).$$

Hence, $r=\frac{(1-(1+i)^{-T})S_0}{\mathrm{Call}(S_0,T)}$.

Example 2

The risk–free effective rate of interest is 7%. The current price of the S & P 500 is 1500. The price of an European call with strike 1500 and expiration date in 3 years is 400. Find the participation rate of a three–year ELCD which provides 100% principal protection.

Example 2

The risk–free effective rate of interest is 7%. The current price of the S & P 500 is 1500. The price of an European call with strike 1500 and expiration date in 3 years is 400. Find the participation rate of a three–year ELCD which provides 100% principal protection.

Solution: We have that

$$r = \frac{(1-(1+i)^{-T})S_0}{\text{Call}(S_0, T)}$$

$$= \frac{(1-(1.07)^{-3})1500}{400} = 0.6888829617 = 68.8829617\%.$$

Suppose that an ELCD return all the principal invested at expiration, offers a guaranteed rate of return g, where $g < i$, and a participation rate r on the S & P 500 stock index. The payoff of this ELCD is

$$P(1+g)^T + Pr \max\left(\frac{S_T - S_0}{S_0}, 0\right) = P(1+g)^T + \frac{Pr}{S_0}\max(S_T - S_0, 0).$$

We can get this payoff by buying a zero–coupon bond with face value $P(1+g)^T$ and a S_0–strike call option for $\frac{Pr}{S_0}$ shares. The cost of this portfolio is

$$P(1+g)^T(1+i)^{-T} + \frac{Pr}{S_0}\text{Call}(S_0, T).$$

If there exist no arbitrage,

$$P = P(1+g)^T(1+i)^{-T} + \frac{Pr}{S_0}\text{Call}(S_0, T)$$

and

$$r = \frac{(1-(1+g)^T(1+i)^{-T})S_0}{\text{Call}(S_0, T)}.$$

Example 3

The risk–free effective rate of interest is 5%. *The current price of the S & P 500 is 1500. The price of an European call with strike 1500 and expiration date in six month is 125. Find the participation rate of a ELCD which provides 100% principal protection and a guaranteed annual interest rate of* 1.5%.

Example 3

The risk–free effective rate of interest is 5%. *The current price of the S & P 500 is 1500. The price of an European call with strike 1500 and expiration date in six month is 125. Find the participation rate of a ELCD which provides 100% principal protection and a guaranteed annual interest rate of* 1.5%.

Solution:

$$r = \frac{(1-(1+g)^T(1+i)^{-T})S_0}{\text{Call}(S_0,T)}$$

$$= \frac{(1-(1.015)^{0.5}(1.05)^{-0.5})1500}{125} = 0.201695 = 20.1695\%.$$

