Manual for SOA Exam FM/CAS Exam 2.

Chapter 7. Derivatives markets.

Section 7.8. Spreads.

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Spreads

An **option spread** (or a **vertical spread**) is a combination of only calls or only puts, in which some options are bought and some others are sold. By buying/selling several call/puts we can create portfolios useful for many different objectives. A **ratio spread** is a combination of buying m calls at one strike price and selling n calls at a different strike price.

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Speculating on the increase of an asset price. Bull spread.

Definition 1

A **bull spread** *consists on buying a* K_1*–strike call and selling a* K_2*–strike call, both with the same expiration date* T *and nominal amount, where* $0 < K_1 < K_2$*.*

A way to speculate on the increase of an asset price is buying the asset. This position needs a lot of investment. Another way to speculate on the increase of an asset price is to buy a call option. A bull spread allows to speculate on increase of an asset price by making a limited investment.

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The payoff for buying a K_1–strike call is $\max(S_T - K_1, 0)$.
The payoff for selling a K_2–strike call is $-\max(S_T - K_2, 0)$.

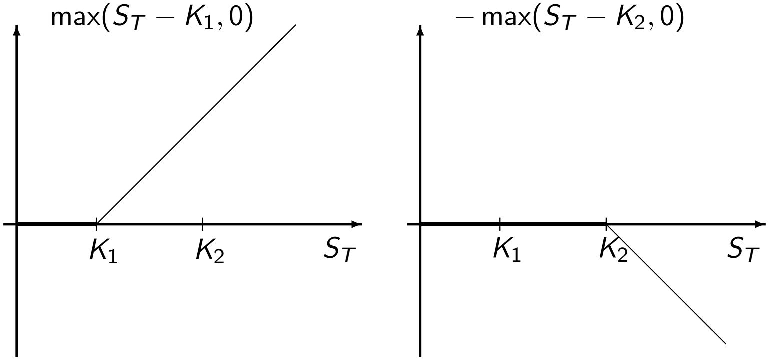

Figure 1: Payoff of the calls in a bull spread

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The bull spread payoff is

$$
\begin{aligned}
&\max(S_T - K_1, 0) - \max(S_T - K_2, 0)\\
=&\max(S_T - K_1, 0) + K_2 - K_1 - \max(S_T - K_1, K_2 - K_1)\\
=&\max(S_T - K_1, 0) + K_2 - K_1 - \max(S_T - K_1, 0, K_2 - K_1)\\
=&\max(S_T - K_1, 0) + K_2 - K_1 - \max(\max(S_T - K_1, 0), K_2 - K_1)\\
=&\min(\max(S_T - K_1, 0), K_2 - K_1)\\
=&\begin{cases} 0 & \text{if } S_T < K_1,\\ S_T - K_1 & \text{if } K_1 \le S_T < K_2,\\ K_2 - K_1 & \text{if } K_2 \le S_T. \end{cases}
\end{aligned}
$$

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$$\max(S_T - K_1, 0) - \max(S_T - K_2, 0)$$

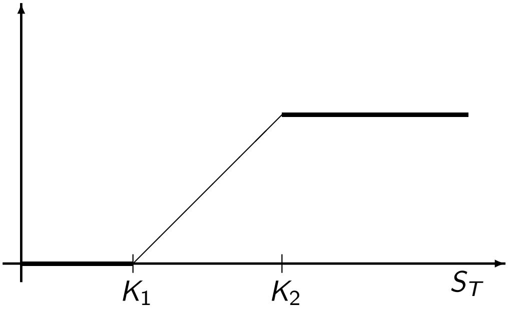

Figure 2: Payoff of a bull spread

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The profit of a bull spread is

$$
\begin{aligned}
&\min(\max(S_T - K_1, 0), K_2 - K_1) \\
&- (\mathrm{Call}(K_1, T) - \mathrm{Call}(K_2, T))(1+i)^T \\
=&\begin{cases}
-(\mathrm{Call}(K_1, T) - \mathrm{Call}(K_2, T))(1+i)^T & \text{if } S_T < K_1, \\
S_T - K_1 - (\mathrm{Call}(K_1, T) - \mathrm{Call}(K_2, T))(1+i)^T & \text{if } K_1 \le S_T < K_2, \\
K_2 - K_1 - (\mathrm{Call}(K_1, T) - \mathrm{Call}(K_2, T))(1+i)^T & \text{if } K_2 \le S_T.
\end{cases}
\end{aligned}
$$

Figure 3 shows a graph of the profit of a bull spread. Notice that the profit is positive for values of S_T large enough.

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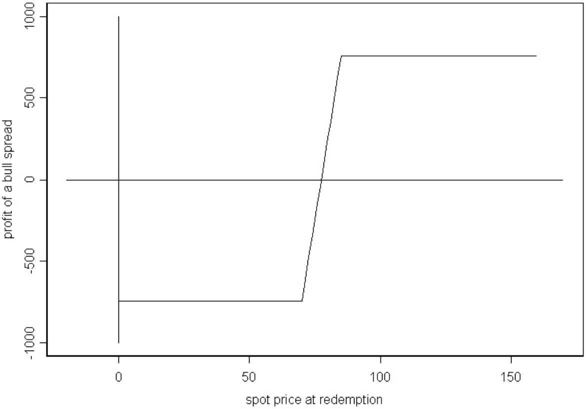

Figure 3: Profit for a bull spread.

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In this section, we will use the values of calls/puts in Table 1.

Table 1: Prices of some calls and some puts.

K	65	70	75	80	85
$\mathrm{Call}(K, T)$	14.31722	10.75552	7.78971	5.444947	3.680736
$\mathrm{Put}(K, T)$	1.221977	2.422184	4.218281	6.635423	9.633117

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Example 1

(Use Table 1) Ronald buys a \$70–strike call and sells a \$85–strike call for 100 shares of XYZ stock. Both have expiration date one year from now. The current price of one share of XYZ stock is \$75. The risk free annual effective rate of interest is 5%. *The premium per share of \$70–strike and \$85–strike calls are 10.75552 and 3.680736 respectively.*

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Example 1

(Use Table 1) Ronald buys a \$70–strike call and sells a \$85–strike call for 100 shares of XYZ stock. Both have expiration date one year from now. The current price of one share of XYZ stock is \$75. The risk free annual effective rate of interest is 5%*. The premium per share of \$70–strike and \$85–strike calls are 10.75552 and 3.680736 respectively.*

(i) Find the profit at expiration as a function of the strike price.

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Example 1

(Use Table 1) Ronald buys a \$70–strike call and sells a \$85–strike call for 100 shares of XYZ stock. Both have expiration date one year from now. The current price of one share of XYZ stock is \$75. The risk free annual effective rate of interest is 5%. *The premium per share of \$70–strike and \$85–strike calls are 10.75552 and 3.680736 respectively.*

(i) Find the profit at expiration as a function of the strike price.

Solution: (i) Ronald's profit is

$$
\begin{aligned}
&(100)\left(\min(\max(S_T-70,0),85-70)-(10.75552-3.680736)(1.05)\right)\\
=&(100)\left(\min(\max(S_T-70,0),85-70)-7.428523\right)\\
=&\begin{cases} -(100)(7.428523) & \text{if } S_T<70,\\ (100)(S_T-70-7.428523) & \text{if } 70\le S_T<85,\\ (100)(85-70-7.428523) & \text{if } 85\le S_T. \end{cases}
\end{aligned}
$$

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Example 1

(Use Table 1) Ronald buys a \$70–strike call and sells a \$85–strike call for 100 shares of XYZ stock. Both have expiration date one year from now. The current price of one share of XYZ stock is \$75. The risk free annual effective rate of interest is 5%*. The premium per share of \$70–strike and \$85–strike calls are 10.75552 and 3.680736 respectively.*

(ii) Make a table with Ronald's profit when the spot price at expiration is \$65, \$70, \$75, \$80, \$85, \$90.

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Example 1

(Use Table 1) Ronald buys a \$70–strike call and sells a \$85–strike call for 100 shares of XYZ stock. Both have expiration date one year from now. The current price of one share of XYZ stock is \$75. The risk free annual effective rate of interest is 5%. The premium per share of \$70–strike and \$85–strike calls are 10.75552 and 3.680736 respectively.

(ii) Make a table with Ronald's profit when the spot price at expiration is \$65, \$70, \$75, \$80, \$85, \$90.

Solution: (ii)

Spot Price	65	70	75
Ronald's profit	−742.8523	−742.8523	−242.8523

Spot Price	80	85	90
Ronald's profit	257.1477	757.1477	757.1477

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Example 1

(Use Table 1) Ronald buys a \$70–strike call and sells a \$85–strike call for 100 shares of XYZ stock. Both have expiration date one year from now. The current price of one share of XYZ stock is \$75. The risk free annual effective rate of interest is 5%. *The premium per share of \$70–strike and \$85–strike calls are 10.75552 and 3.680736 respectively.*

(iii) Draw the graph of Ronald's profit.

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Example 1

(Use Table 1) Ronald buys a \$70–strike call and sells a \$85–strike call for 100 shares of XYZ stock. Both have expiration date one year from now. The current price of one share of XYZ stock is \$75. The risk free annual effective rate of interest is 5%. *The premium per share of \$70–strike and \$85–strike calls are 10.75552 and 3.680736 respectively.*

(iii) Draw the graph of Ronald's profit.

Solution: (ii) The graph of the profit is Figure 3.

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Let $0 < K_1 < K_2$. We have that

$$\begin{aligned}
&[\text{long } K_1 - \text{strike call }] + [\text{short } K_1 - \text{strike put }] \\
&\equiv [\text{buying asset for } K_1 \text{ at time } T], \\
&[\text{long } K_2 - \text{strike call }] + [\text{short } K_2 - \text{strike put }] \\
&\equiv [\text{buying asset for } K_2 \text{ at time } T].
\end{aligned}$$

Since the two investment strategies differ by a constant, they have the same profit. Hence, so do the following strategies,

$$\begin{aligned}
&[\text{long } K_1 - \text{strike call }] + [\text{short } K_2 - \text{strike call }], \\
&[\text{long } K_1 - \text{strike put }] + [\text{short } K_2 - \text{strike put }].
\end{aligned}$$

In other words, buying a K_1–strike call and selling a K_2–strike call has the same profit as buying a K_1–strike put and selling a K_2–strike put.

We can form a bull spread either buying a K_1–strike call and selling a K_2–strike call, or buying a K_1–strike put and selling a K_2–strike put.

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Example 2

The current price of XYZ stock is \$75 per share. The effective annual interest rate is 5%. *Elizabeth, Daniel and Catherine believe that the price of XYZ stock is going to appreciate significantly in the next year. Each person has \$10000 to invest. The premium of a one–year* 85*–strike call option is* 3.680736 *per share. The premium of a one–year* 75*–strike call option is* 7.78971 *per share. Elizabeth buys a one–year zero–coupon bond for \$*10000*. She also enters into a one–year forward contract on XYZ stock worth equal to the her bond payoff at redemption. Daniel buys a one–year* 85*–strike call option which costs \$*10000*. Catherine buys a one–year* 75*–strike call option and sells a one–year* 85*–strike call option. The nominal amounts on both calls are the same. The difference between the cost of the* 85*–strike call option and the* 75*–strike call option is 10000. Suppose that the stock price at redemption is 90 per share. Calculate the profits and the yield rates for Elizabeth, Daniel and Catherine. Which one makes a bigger profit?*

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Solution: The price of a long forward is $75(1.05) = 78.75$. So, Elizabeth long forward is for $\frac{(10000)(1.05)}{78.75} = 133.333333$ shares. Elizabeth's profit is $133.333333(90 - 78.75) = 1500$. Her yield rate is $\frac{1500}{10000} = 15\%$.

The nominal amount in Daniel's call is $\frac{10000}{3.680736} = 2716.847935$ shares. Daniel's profit is $2716.847935(90 - 85) = 13584.24$.

Daniel's yield of return is $\frac{13584.24}{10000} = 135.8424\%$.

The nominal amount in Catherine's calls is $\frac{10000}{7.78971-3.680736} = 2433.697561$ shares. Catherine's profit is $2433.697561(85 - 75) = 24336.97561$. Catherine's yield of return is $\frac{24336.97561}{10000} = 243.3697561\%$.

Catherine's profit is biggest.

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Speculating on the decrease of an asset price. Bear spread.

A **bear spread** is precisely the opposite of a bull spread. Suppose that you want to speculate on the price of an asset decreasing. Let $0 < K_1 < K_2$. Consider selling a K_1–strike call and buying a K_2–strike call, both with the same expiration date T. The profit is

$$-\min(\max(S_T - K_1, 0), K_2 - K_1) + (\mathrm{Call}(K_1, T) - \mathrm{Call}(K_2, T))(1+i)^T$$

or

$$\begin{cases} (\mathrm{Call}(K_1, T) - \mathrm{Call}(K_2, T))(1+i)^T & \text{if } S_T < K_1, \\ K_1 - S_T + (\mathrm{Call}(K_1, T) - \mathrm{Call}(K_2, T))(1+i)^T & \text{if } K_1 \le S_T < K_2, \\ K_1 - K_2 + (\mathrm{Call}(K_1, T) - \mathrm{Call}(K_2, T))(1+i)^T & \text{if } K_2 \le S_T. \end{cases}$$

A graph of this profit is in Figure 4. Notice that the profit is positive for values of S_T small enough.

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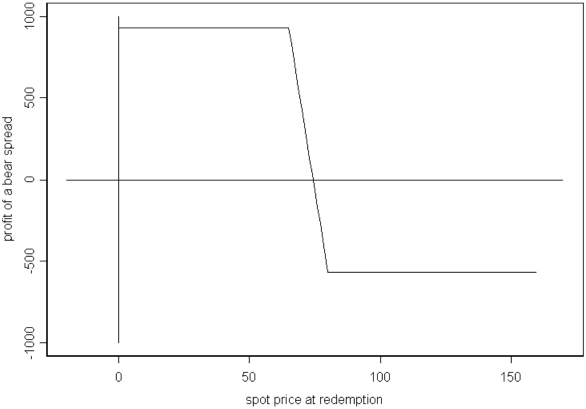

Figure 4: Profit for a bear spread.

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Example 3

(Use Table 1) Rebecca sells a \$65–strike call and buys a \$80–strike call for 1000 shares of XYZ stock. Both have expiration date one year from now. The current price of one share of XYZ stock is \$75. The risk free annual effective rate of interest is 5%. *The premium per share of \$65–strike and \$80–strike calls are 14.31722 and 5.444947 respectively.*

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Example 3

(Use Table 1) Rebecca sells a \$65–strike call and buys a \$80–strike call for 1000 shares of XYZ stock. Both have expiration date one year from now. The current price of one share of XYZ stock is \$75. The risk free annual effective rate of interest is 5%. *The premium per share of \$65–strike and \$80–strike calls are 14.31722 and 5.444947 respectively.*

(i) Find Rebecca's profit as a function of the spot price at expiration.

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Example 3

(Use Table 1) Rebecca sells a \$65–strike call and buys a \$80–strike call for 1000 shares of XYZ stock. Both have expiration date one year from now. The current price of one share of XYZ stock is \$75. The risk free annual effective rate of interest is 5%. *The premium per share of \$65–strike and \$80–strike calls are 14.31722 and 5.444947 respectively.*

(i) Find Rebecca's profit as a function of the spot price at expiration.

Solution: (i) Rebecca's profit is

$$\begin{aligned}
&-(100)\max(S_T-65,0)+(100)\max(S_T-80,0)\\
&+(100)(14.31722-5.444947)(1.05)\\
=&-(100)\min(\max(S_T-65,0),80-65)+(100)(9.31588665)\\
=&\begin{cases}(100)(9.31588665) & \text{if } S_T<65\\ (100)(65-S_T+9.31588665) & \text{if } 65\le S_T<80\\ (100)(65-80+9.31588665) & \text{if } 80\le S_T.\end{cases}
\end{aligned}$$

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Example 3

(Use Table 1) Rebecca sells a \$65–strike call and buys a \$80–strike call for 1000 shares of XYZ stock. Both have expiration date one year from now. The current price of one share of XYZ stock is \$75. The risk free annual effective rate of interest is 5%*. The premium per share of \$65–strike and \$80–strike calls are 14.31722 and 5.444947 respectively.*

(ii) Make a table with Rebecca's profit when the spot price at expiration is \$60, \$65, \$70, \$75, \$80, \$85, \$90.

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Example 3

(Use Table 1) Rebecca sells a \$65–strike call and buys a \$80–strike call for 1000 shares of XYZ stock. Both have expiration date one year from now. The current price of one share of XYZ stock is \$75. The risk free annual effective rate of interest is 5%. The premium per share of \$65–strike and \$80–strike calls are 14.31722 and 5.444947 respectively.

(ii) Make a table with Rebecca's profit when the spot price at expiration is \$60, \$65, \$70, \$75, \$80, \$85, \$90.

Solution: (ii)

Spot Price	60	65	70	75
Rebecca's profit	931.59	931.59	431.59	−68.41

Spot Price	75	80	85	90
Rebecca's profit	−68.41	−568.41	−568.41	−568.41

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Example 3

(Use Table 1) Rebecca sells a \$65–strike call and buys a \$80–strike call for 1000 shares of XYZ stock. Both have expiration date one year from now. The current price of one share of XYZ stock is \$75. The risk free annual effective rate of interest is 5%. *The premium per share of \$65–strike and \$80–strike calls are 14.31722 and 5.444947 respectively.*

(iii) Draw the graph of Rebecca's profit.

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Example 3

(Use Table 1) Rebecca sells a \$65–strike call and buys a \$80–strike call for 1000 shares of XYZ stock. Both have expiration date one year from now. The current price of one share of XYZ stock is \$75. The risk free annual effective rate of interest is 5%. *The premium per share of \$65–strike and \$80–strike calls are 14.31722 and 5.444947 respectively.*

(iii) Draw the graph of Rebecca's profit.

Solution: (iii) Figure 4 shows the graph of the profit.

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Collar

A **collar** is the purchase of a put option at a strike price and the sale of a call option at a higher strike price. Let K_1 be the strike price of the put option. Let K_2 be the strike price of the call option. Assume that $K_1 < K_2$. The profit of this strategy is

$$
\begin{aligned}
&\max(K_1 - S_T, 0) - \max(S_T - K_2, 0) \\
&- (\mathrm{Put}(K_1, T) - \mathrm{Call}(K_2, T))(1+i)^T \\
=&\begin{cases}
K_1 - S_T - (\mathrm{Put}(K_1, T) - \mathrm{Call}(K_2, T))(1+i)^T & \text{if } S_T < K_1, \\
-(\mathrm{Put}(K_1, T) - \mathrm{Call}(K_2, T))(1+i)^T & \text{if } K_1 \le S_T < K_2, \\
K_2 - S_T - (\mathrm{Put}(K_1, T) - \mathrm{Call}(K_2, T))(1+i)^T & \text{if } K_2 \le S_T.
\end{cases}
\end{aligned}
$$

A collar can be use to speculate on the decrease of price of an asset.

The collar width is the difference between the call strike and the put strike.

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Example 4

(Use Table 1) Toto buys a \$65–strike put option and sells a \$80–strike call for 100 shares of XYZ stock. Both have expiration date one year from now. The current price of one share of XYZ stock is \$75. The risk free annual effective rate of interest is 5%*. The premium of a \$65–strike put option is 1.221977 per share. The premium of a \$80–strike call option is 5.444947 per share.*

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Example 4

(Use Table 1) Toto buys a \$65–strike put option and sells a \$80–strike call for 100 shares of XYZ stock. Both have expiration date one year from now. The current price of one share of XYZ stock is \$75. The risk free annual effective rate of interest is 5%*.*
The premium of a \$65–strike put option is 1.221977 per share.
The premium of a \$80–strike call option is 5.444947 per share.
(i) Find Toto's profit as a function of the spot price at expiration.

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Example 4

(Use Table 1) Toto buys a \$65–strike put option and sells a \$80–strike call for 100 shares of XYZ stock. Both have expiration date one year from now. The current price of one share of XYZ stock is \$75. The risk free annual effective rate of interest is 5%. The premium of a \$65–strike put option is 1.221977 per share. The premium of a \$80–strike call option is 5.444947 per share.

(i) Find Toto's profit as a function of the spot price at expiration.

Solution: (i) Toto's profit is

$$\begin{aligned}
&(100)\left(\max(65-S_T,0)-\max(S_T-80,0)\right.\\
&\left.-(1.221977-5.444947)(1.05)\right)\\
=&(100)\left(\max(65-S_T,0)-\max(S_T-80,0)+4.4341185\right)\\
=&\begin{cases}(100)(65-S_T+4.4341185) & \text{if } S_T<65,\\ 100(4.4341185) & \text{if } 65\le S_T<80,\\ 100(80-S_T+4.4341185) & \text{if } 80\le S_T.\end{cases}
\end{aligned}$$

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Example 4

(Use Table 1) Toto buys a \$65–strike put option and sells a \$80–strike call for 100 shares of XYZ stock. Both have expiration date one year from now. The current price of one share of XYZ stock is \$75. The risk free annual effective rate of interest is 5%. The premium of a \$65–strike put option is 1.221977 per share. The premium of a \$80–strike call option is 5.444947 per share.

(ii) Make a table with Toto's profit when the spot price at expiration is \$55, \$60, \$65, \$70, \$75, \$80, \$85.

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Example 4

(Use Table 1) Toto buys a \$65–strike put option and sells a \$80–strike call for 100 shares of XYZ stock. Both have expiration date one year from now. The current price of one share of XYZ stock is \$75. The risk free annual effective rate of interest is 5%. The premium of a \$65–strike put option is 1.221977 per share. The premium of a \$80–strike call option is 5.444947 per share.

(ii) Make a table with Toto's profit when the spot price at expiration is \$55, \$60, \$65, \$70, \$75, \$80, \$85.

Solution: (ii) A table with Toto's profit is

Spot Price	55	60	65	70
Toto's profit	1443.41	943.41	443.41	443.41

Spot Price	75	80	85	90
Toto's profit	443.41	443.41	−56.59	−556.59

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Example 4

(Use Table 1) Toto buys a \$65–strike put option and sells a \$80–strike call for 100 shares of XYZ stock. Both have expiration date one year from now. The current price of one share of XYZ stock is \$75. The risk free annual effective rate of interest is 5%. *The premium of a \$65–strike put option is 1.221977 per share. The premium of a \$80–strike call option is 5.444947 per share.*

(iii) Draw the graph of Toto's profit.

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Example 4

(Use Table 1) Toto buys a \$65–strike put option and sells a \$80–strike call for 100 shares of XYZ stock. Both have expiration date one year from now. The current price of one share of XYZ stock is \$75. The risk free annual effective rate of interest is 5%. The premium of a \$65–strike put option is 1.221977 per share. The premium of a \$80–strike call option is 5.444947 per share.

(iii) Draw the graph of Toto's profit.

Solution: (iii) The graph of the profit is on Figure 5.

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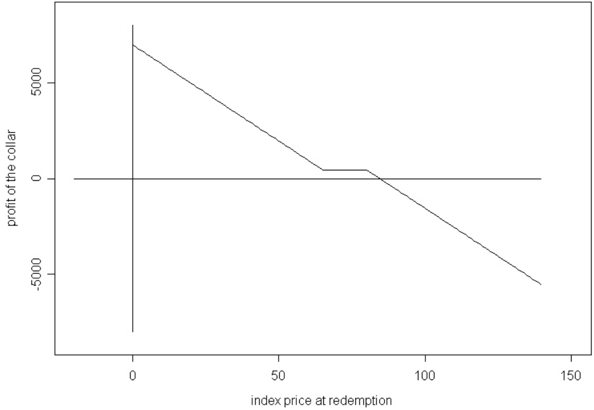

Figure 5: Profit for a collar.

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Written collar

A **written collar** is a reverse collar.

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Collars are used to insure a long position on a stock. This position is called a **collared stock**. A collared stock involves buying the index, buy a K_1–strike put option and selling a K_2–strike call option, where $K_1 < K_2$. The payoff per share of this strategy is

$$\begin{aligned} & S_T + \max(K_1 - S_T, 0) - \max(S_T - K_2, 0) \\ = & \max(K_1, S_T) - \max(S_T, K_2) + K_2 \\ = & \max(K_1, S_T) - \max(S_T, K_1, K_2) + K_2 \\ = & \min(\max(K_1, S_T), K_2) \end{aligned}$$

The profit per share of this portfolio is

$$\begin{aligned} & \min(\max(K_1, S_T), K_2) - (S_0 + \mathrm{Put}(K_1, T) - \mathrm{Call}(K_2, T))(1+i)^T \\ = & \begin{cases} K_1 - (S_0 + \mathrm{Put}(K_1, T) - \mathrm{Call}(K_2, T))(1+i)^T & \text{if } S_T < K_1, \\ S_T - (S_0 + \mathrm{Put}(K_1, T) - \mathrm{Call}(K_2, T))(1+i)^T & \text{if } K_1 \le S_T < K_2, \\ K_2 - (S_0 + \mathrm{Put}(K_1, T) - \mathrm{Call}(K_2, T))(1+i)^T & \text{if } K_2 \le S_T. \end{cases} \end{aligned}$$

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Example 5

(Use Table 1) Maggie buys 100 shares of XYZ stock, a \$65–strike put option and sells a \$80–strike call. Both options are for 100 shares of XYZ stock and have expiration date one year from now. The current price of one share of XYZ stock is \$75. The risk free annual effective rate of interest is 5%. *The premium per share of a \$65–strike put option is* 1.221977. *The premium per share of a \$80–strike call is* 5.444947.

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Example 5

(Use Table 1) Maggie buys 100 shares of XYZ stock, a \$65–strike put option and sells a \$80–strike call. Both options are for 100 shares of XYZ stock and have expiration date one year from now. The current price of one share of XYZ stock is \$75. The risk free annual effective rate of interest is 5%*. The premium per share of a \$65–strike put option is* 1.221977*. The premium per share of a \$80–strike call is* 5.444947*.*

(i) Find Maggie's profit as a function of the spot price at expiration.

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Example 5

(Use Table 1) Maggie buys 100 shares of XYZ stock, a \$65–strike put option and sells a \$80–strike call. Both options are for 100 shares of XYZ stock and have expiration date one year from now. The current price of one share of XYZ stock is \$75. The risk free annual effective rate of interest is 5%. *The premium per share of a \$65–strike put option is* 1.221977. *The premium per share of a \$80–strike call is* 5.444947.

(i) Find Maggie's profit as a function of the spot price at expiration.

Solution: (i) Maggie's profit is

$$
\begin{aligned}
&(100)\left(\min(\max(65, S_T), 80) - (75 + 1.221977 - 5.444947)(1.05)^1\right) \\
=&(100)\left(\min(\max(65, S_T), 80) - 74.315882\right) \\
=&\begin{cases} 100(65 - 74.315882) & \text{if } S_T < 65, \\ 100(S_T - 74.315882) & \text{if } 65 \le S_T < 80, \\ 100(80 - 74.315882) & \text{if } 80 \le S_T. \end{cases}
\end{aligned}
$$

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Example 5

(Use Table 1) Maggie buys 100 shares of XYZ stock, a \$65–strike put option and sells a \$80–strike call. Both options are for 100 shares of XYZ stock and have expiration date one year from now. The current price of one share of XYZ stock is \$75. The risk free annual effective rate of interest is 5%. *The premium per share of a \$65–strike put option is* 1.221977. *The premium per share of a \$80–strike call is* 5.444947.

(ii) Make a table with Maggie's profit when the spot price at expiration is \$60, \$65, \$70, \$75, \$80, \$85.

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Example 5

(Use Table 1) Maggie buys 100 shares of XYZ stock, a \$65–strike put option and sells a \$80–strike call. Both options are for 100 shares of XYZ stock and have expiration date one year from now. The current price of one share of XYZ stock is \$75. The risk free annual effective rate of interest is 5%*. The premium per share of a \$65–strike put option is* 1.221977*. The premium per share of a \$80–strike call is* 5.444947*.*

(ii) Make a table with Maggie's profit when the spot price at expiration is \$60, \$65, \$70, \$75, \$80, \$85.

Solution: (ii) A table with Maggie's profit for the considered spot prices is

Spot Price	60	65	70
Maggie's profit	−931.59	−931.59	−431.59

Spot Price	75	80	85
Maggie's profit	68.41	568.41	568.41

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Example 5

(Use Table 1) Maggie buys 100 shares of XYZ stock, a \$65–strike put option and sells a \$80–strike call. Both options are for 100 shares of XYZ stock and have expiration date one year from now. The current price of one share of XYZ stock is \$75. The risk free annual effective rate of interest is 5%. *The premium per share of a \$65–strike put option is* 1.221977. *The premium per share of a \$80–strike call is* 5.444947.

(iii) Draw the graph of Maggie's profit.

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Example 5

(Use Table 1) Maggie buys 100 shares of XYZ stock, a \$65–strike put option and sells a \$80–strike call. Both options are for 100 shares of XYZ stock and have expiration date one year from now. The current price of one share of XYZ stock is \$75. The risk free annual effective rate of interest is 5%. *The premium per share of a \$65–strike put option is* 1.221977. *The premium per share of a \$80–strike call is* 5.444947.

(iii) Draw the graph of Maggie's profit.

Solution: (iii) The graph of the profit is on Figure 6.

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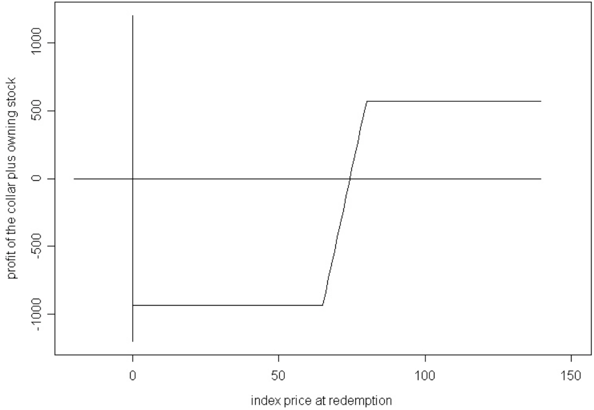

Figure 6: Profit for a collar plus owning the stock.

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Suppose that you worked five years for Microsoft and have \$100000 in stock of this company. You would like to insure this position buying a collar with expiration T years from now. You can choose K_1 and K_2 so that the combined premium is zero. In this case, no matter what the price of the stock T years form now, you will have \$100000 or more. The cost of buying this insurance is zero. Notice that you have not got anything from free, you have loss interest in the stock. You also can make a collar with premium zero, by taking $K_1 = K_2 = F_{0,T}$. In this case you will receive $F_{0,T}$ at time T, i.e. you are entering into a synthetic forward.
Suppose that a zero–cost collar consists of buying a K_1–strike put option and selling a K_2–strike call option, where $K_1 < K_2$. The profit of a zero–cost collar is

$$\max(K_1 - S_T, 0) - \max(S_T - K_2, 0) = \begin{cases} K_1 - S_T & \text{if } S_T < K_1, \\ 0 & \text{if } K_1 \le S_T < K_2, \\ K_2 - S_T & \text{if } K_2 \le S_T. \end{cases}$$

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Speculating on volatility. Straddle

A **straddle** consists of buying a K–strike call and a K–strike put with the same time to expiration. The payoff of this strategy is

$$\begin{aligned} & \max(K - S_T, 0) + \max(S_T - K, 0) \\ = & \max(K - S_T, 0) - \min(K - S_T, 0) \\ = & |S_T - K|. \end{aligned}$$

Its profit is

$$|S_T - K| - (\mathrm{Put}(K, T) + \mathrm{Call}(K, T))(1 + i)^T.$$

A straddle is used to bet that the volatility of the market is higher than the market's assessment of volatility. Notice that the prices of the put and call use the market's assessment of volatility.

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Example 6

(Use Table 1) Pam buys a \$80–strike call option and a \$80–strike put for 100 shares of XYZ stock. Both have expiration date one year from now. The current price of one share of XYZ stock is \$75. The risk free annual effective rate of interest is 5%*. The premium per share of a \$80–strike call option is 5.444947. The premium per share of a \$80–strike put option is 6.635423.*

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Example 6

(Use Table 1) Pam buys a \$80–strike call option and a \$80–strike put for 100 shares of XYZ stock. Both have expiration date one year from now. The current price of one share of XYZ stock is \$75. The risk free annual effective rate of interest is 5%. The premium per share of a \$80–strike call option is 5.444947. The premium per share of a \$80–strike put option is 6.635423.

(i) Calculate Pam's profit as a function of the spot price at expiration.

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Example 6

(Use Table 1) Pam buys a \$80–strike call option and a \$80–strike put for 100 shares of XYZ stock. Both have expiration date one year from now. The current price of one share of XYZ stock is \$75. The risk free annual effective rate of interest is 5%. The premium per share of a \$80–strike call option is 5.444947. The premium per share of a \$80–strike put option is 6.635423.

(i) Calculate Pam's profit as a function of the spot price at expiration.

Solution: (i) The future value per share of the cost of entering the option contracts is

$$\begin{aligned}&(\mathrm{Call}(80, T) + \mathrm{Put}(80, T))(1+i)^T\\=&(5.444947 + 6.635423)(1.05) = 12.6843885.\end{aligned}$$

Pam's profit is

$$(100)(|S_T - 80| - 12.6843885).$$

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Example 6

(Use Table 1) Pam buys a \$80–strike call option and a \$80–strike put for 100 shares of XYZ stock. Both have expiration date one year from now. The current price of one share of XYZ stock is \$75. The risk free annual effective rate of interest is 5%. *The premium per share of a \$80–strike call option is 5.444947. The premium per share of a \$80–strike put option is 6.635423.*

(ii) Make a table with Pam's profit when the spot price at expiration is \$65, \$70, \$75, \$80, \$85, \$90, \$95.

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Example 6

(Use Table 1) Pam buys a \$80–strike call option and a \$80–strike put for 100 shares of XYZ stock. Both have expiration date one year from now. The current price of one share of XYZ stock is \$75. The risk free annual effective rate of interest is 5%. The premium per share of a \$80–strike call option is 5.444947. The premium per share of a \$80–strike put option is 6.635423.

(ii) Make a table with Pam's profit when the spot price at expiration is \$65, \$70, \$75, \$80, \$85, \$90, \$95.

Solution: (ii) Pam's profit table is

Spot Price	65	70	75	80
Pam's profit	231.56	−268.44	−768.44	−1268.44

Spot Price	80	85	90	95
Pam's profit	−1268.44	−768.44	−268.44	231.56

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Example 6

(Use Table 1) Pam buys a \$80–strike call option and a \$80–strike put for 100 shares of XYZ stock. Both have expiration date one year from now. The current price of one share of XYZ stock is \$75. The risk free annual effective rate of interest is 5%. The premium per share of a \$80–strike call option is 5.444947. The premium per share of a \$80–strike put option is 6.635423.

(iii) Draw the graph of Pam's profit.

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Example 6

(Use Table 1) Pam buys a \$80–strike call option and a \$80–strike put for 100 shares of XYZ stock. Both have expiration date one year from now. The current price of one share of XYZ stock is \$75. The risk free annual effective rate of interest is 5%. *The premium per share of a \$80–strike call option is 5.444947. The premium per share of a \$80–strike put option is 6.635423.*

(iii) Draw the graph of Pam's profit.

Solution: (iii) The graph of the profit is on Figure 7.

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Example 6

(Use Table 1) Pam buys a \$80–strike call option and a \$80–strike put for 100 shares of XYZ stock. Both have expiration date one year from now. The current price of one share of XYZ stock is \$75. The risk free annual effective rate of interest is 5%. *The premium per share of a \$80–strike call option is 5.444947. The premium per share of a \$80–strike put option is 6.635423.*

(iv) Find the values of the spot price at expiration at which Pam makes a profit.

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Example 6

(Use Table 1) Pam buys a \$80–strike call option and a \$80–strike put for 100 shares of XYZ stock. Both have expiration date one year from now. The current price of one share of XYZ stock is \$75. The risk free annual effective rate of interest is 5%. The premium per share of a \$80–strike call option is 5.444947. The premium per share of a \$80–strike put option is 6.635423.

(iv) Find the values of the spot price at expiration at which Pam makes a profit.

Solution: (iv) Pam makes a profit if $(100)(|S_T - 80| - 12.6843885) > 0$, i.e. if $|S_T - 80| > 12.6843885$. This can happen if either $S_T - 80 < -12.6843885$, or $S_T - 80 > 12.6843885$. We have that $S_T - 80 < -12.6843885$ is equivalent to $S_T < 80 - 12.6843885 = 67.3156115$. $S_T - 80 > 12.6843885$ is equivalent to $S_T > 92.6843885$. Hence, Pam makes a profit if either $S_T < 67.3156115$ or $S_T > 92.6843885$.

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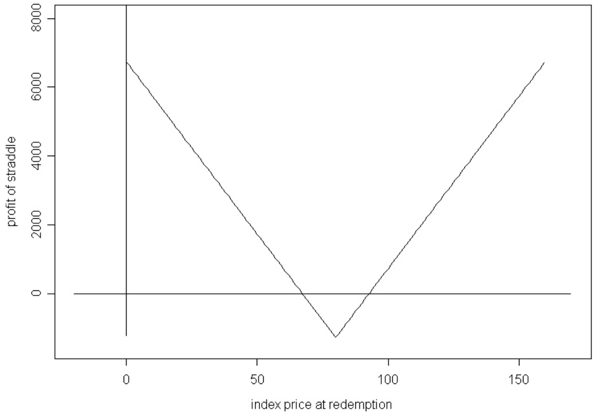

Figure 7: Profit for a straddle.

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Strangle

A **strangle** consists of buying a K_1–strike put and a K_2–strike call with the same expiration date, where $K_1 < K_2$. The profit of this portfolio is

$$\begin{aligned}&\max(K_1 - S_T, 0) + \max(S_T - K_2, 0)\\&-((\mathrm{Put}(K_1, T) + \mathrm{Call}(K_2, T))(1+i)^T\end{aligned}$$

$$=\begin{cases} K_1 - S_T - ((\mathrm{Put}(K_1, T) + \mathrm{Call}(K_2, T))(1+i)^T & \text{if } S_T < K_1,\\ -((\mathrm{Put}(K_1, T) + \mathrm{Call}(K_2, T))(1+i)^T & \text{if } K_1 \le S_T < K_2\\ S_T - K_2 - ((\mathrm{Put}(K_1, T) + \mathrm{Call}(K_2, T))(1+i)^T. & \text{if } K_2 \le S_T.\end{cases}$$

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Example 7

(Use Table 1) Beth buys a \$75–strike put option and a \$85–strike call for 100 shares of XYZ stock. Both have expiration date one year from now. The current price of one share of XYZ stock is \$75. The risk free annual effective rate of interest is 5%. *The premium per share of a \$75–strike put option is* 4.218281. *The premium per share of a \$85–strike call option is* 3.680736.

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Example 7

(Use Table 1) Beth buys a \$75–strike put option and a \$85–strike call for 100 shares of XYZ stock. Both have expiration date one year from now. The current price of one share of XYZ stock is \$75. The risk free annual effective rate of interest is 5%. *The premium per share of a \$75–strike put option is* 4.218281. *The premium per share of a \$85–strike call option is* 3.680736.

(i) Calculate Beth's profit as a function of S_T.

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Example 7

(Use Table 1) Beth buys a \$75–strike put option and a \$85–strike call for 100 shares of XYZ stock. Both have expiration date one year from now. The current price of one share of XYZ stock is \$75. The risk free annual effective rate of interest is 5%. *The premium per share of a \$75–strike put option is* 4.218281. *The premium per share of a \$85–strike call option is* 3.680736.

(i) Calculate Beth's profit as a function of S_T.

Solution: (i) Beth's profit is

$$\begin{aligned}
&100(\max(75 - S_T, 0) + \max(S_T - 85, 0)\\
&\quad - (4.218281 + 3.680736)(1.05))\\
=&100(\max(75 - S_T, 0) + \max(S_T - 85, 0) - 8.29396785)\\
=&\begin{cases} 100(75 - S_T - 8.29396785) & \text{if } S_T < 75, \\ 100(-8.29396785) & \text{if } 75 \le S_T < 85, \\ 100(S_T - 85 - 8.29396785) & \text{if } 85 \le S_T. \end{cases}
\end{aligned}$$

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Example 7

(Use Table 1) Beth buys a \$75–strike put option and a \$85–strike call for 100 shares of XYZ stock. Both have expiration date one year from now. The current price of one share of XYZ stock is \$75. The risk free annual effective rate of interest is 5%. *The premium per share of a \$75–strike put option is* 4.218281. *The premium per share of a \$85–strike call option is* 3.680736.

(ii) Make a table with Beth's profit when the spot price at expiration is \$50, \$60, \$70, \$80, \$90, \$100, \$110.

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Example 7

(Use Table 1) Beth buys a \$75–strike put option and a \$85–strike call for 100 shares of XYZ stock. Both have expiration date one year from now. The current price of one share of XYZ stock is \$75. The risk free annual effective rate of interest is 5%. *The premium per share of a \$75–strike put option is* 4.218281. *The premium per share of a \$85–strike call option is* 3.680736.

(ii) Make a table with Beth's profit when the spot price at expiration is \$50, \$60, \$70, \$80, \$90, \$100, \$110.

Solution: (ii)

Spot Price	50	60	70	80
Beth's profit	1670.60	670.60	−329.40	−829.40

Spot Price	90	100	110
Beth's profit	−329.40	670.60	1670.60

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Example 7

(Use Table 1) Beth buys a \$75–strike put option and a \$85–strike call for 100 shares of XYZ stock. Both have expiration date one year from now. The current price of one share of XYZ stock is \$75. The risk free annual effective rate of interest is 5%. *The premium per share of a \$75–strike put option is* 4.218281. *The premium per share of a \$85–strike call option is* 3.680736.

(iii) Draw the graph of Beth's profit.

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Example 7

(Use Table 1) Beth buys a \$75–strike put option and a \$85–strike call for 100 shares of XYZ stock. Both have expiration date one year from now. The current price of one share of XYZ stock is \$75. The risk free annual effective rate of interest is 5%. *The premium per share of a \$75–strike put option is* 4.218281. *The premium per share of a \$85–strike call option is* 3.680736.

(iii) Draw the graph of Beth's profit.

Solution: (iii) The graph of the profit is Figure 8.

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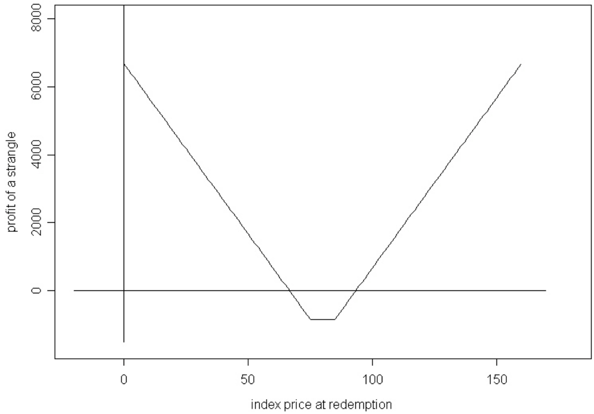

Figure 8: Profit for a strangle.

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The straddle and the strangle bet in volatility of the market in a similar way. Suppose that a straddle and a strangle are centered around the same strike price. The maximum loss of the strangle is smaller than the maximum loss of the straddle. However, the strangle needs more volatility to attain a profit. The possible profit of the strangle is smaller than that of the straddle. See Figure 9.

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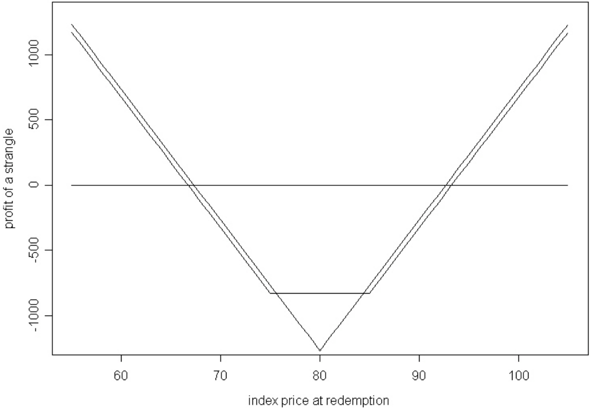

Figure 9: Profit for a straddle and a strangle.

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Written strangle

A **written strangle** consists of selling a K_1–strike call and a K_2–strike put with the same time to expiration, where $0 < K_1 < K_2$. A written strangle is a bet on low volatility.

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Speculating on low volatility: Butterfly spread

Given $0 < K_1 < K_2 < K_3$, a **butterfly spread** consists of:
(i) selling a K_2–strike call and a K_2–strike put, all options for the notional amount.
(ii) buying a K_1–strike call and a K_3–strike put, all options for the notional amount.
The profit per share of this strategy is

$$\begin{aligned}&\max(S_T - K_1, 0) + \max(K_3 - S_T, 0)\\&- \max(S_T - K_2, 0) - \max(K_2 - S_T, 0) - \mathrm{FVP},\end{aligned}$$

where

$$\begin{aligned}\mathrm{FVP} = &\,(\mathrm{Call}(K_1, T) - \mathrm{Call}(K_2, T)\\&-\mathrm{Put}(K_2, T) + \mathrm{Put}(K_3, T))\,(1+i)^T.\end{aligned}$$

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The profit per share of a butterfly spread is

$$
\begin{aligned}
&\max(S_T - K_1, 0) + \max(K_3 - S_T, 0) \\
&- \max(S_T - K_2, 0) - \max(K_2 - S_T, 0) - \mathrm{FVP} \\
=&\begin{cases}
K_3 - K_2 - \mathrm{FVP} & \text{if } S_T < K_1, \\
S_T - K_1 - K_2 + K_3 - \mathrm{FVP} & \text{if } K_1 \le S_T < K_2, \\
-S_T - K_1 + K_2 + K_3 - \mathrm{FVP} & \text{if } K_2 \le S_T < K_3, \\
K_2 - K_1 - \mathrm{FVP} & \text{if } K_3 \le S_T,
\end{cases}
\end{aligned}
$$

The graph of this profit is Figure 10

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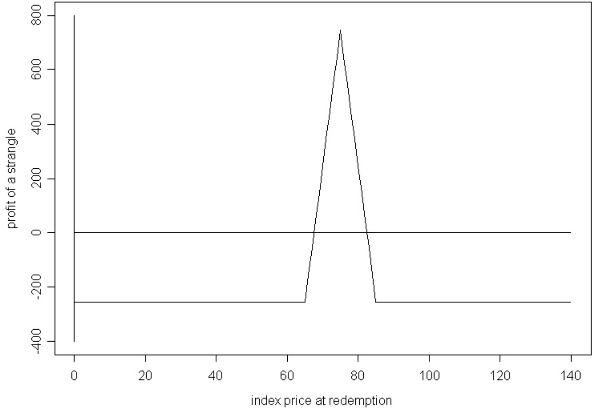

Figure 10: Profit for a butterfly.

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Example 8

(Use Table 1) Steve buys a 65*–strike call and a* 85*–strike put and sells a* 75*–strike call and a* 75*–strike put. All are for 100 shares and have expiration date one year from now. The current price of one share of XYZ stock is \$75. The risk free annual effective rate of interest is* 5%*.*

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Example 8

(Use Table 1) Steve buys a 65*–strike call and a* 85*–strike put and sells a* 75*–strike call and a* 75*–strike put. All are for 100 shares and have expiration date one year from now. The current price of one share of XYZ stock is \$75. The risk free annual effective rate of interest is* 5%*.*

(i) Find Steve's profit as a function of the spot price at expiration.

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Example 8

(Use Table 1) Steve buys a 65–strike call and a 85–strike put and sells a 75–strike call and a 75–strike put. All are for 100 shares and have expiration date one year from now. The current price of one share of XYZ stock is \$75. The risk free annual effective rate of interest is 5%.

(i) Find Steve's profit as a function of the spot price at expiration.

Solution: (i) We have that

$$\begin{aligned}&(\mathrm{Call}(K_1, T) - \mathrm{Call}(K_2, T) - \mathrm{Put}(K_2, T) + \mathrm{Put}(K_3, T))(1+i)^T\\ =&(14.31722 - 9.633117 - 7.78971 + 4.218281)(1.05) = 12.539459.\end{aligned}$$

Steve's profit is

$$\begin{aligned}&(100)\max(S_T - 65, 0) + (100)\max(85 - S_T, 0)\\ &- (100)\max(S_T - 75, 0) - (100)\max(75 - S_T, 0) - (100)(12.539459)\end{aligned}$$

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Example 8

(Use Table 1) Steve buys a 65*–strike call and a* 85*–strike put and sells a* 75*–strike call and a* 75*–strike put. All are for 100 shares and have expiration date one year from now. The current price of one share of XYZ stock is $75. The risk free annual effective rate of interest is* 5%.

(i) Find Steve's profit as a function of the spot price at expiration.

Solution: (i) (continuation)

$$
\begin{aligned}
&(100)\max(S_T - 65, 0) + (100)\max(85 - S_T, 0)\\
&-(100)\max(S_T - 75, 0) - (100)\max(75 - S_T, 0) - (100)(12.539459)
\end{aligned}
$$

$$
= \begin{cases}
100(10 - 12.539459) & \text{if } S_T < 65,\\
100(S_T - 55 - 12.539459) & \text{if } 65 \le S_T < 75,\\
100(-S_T + 95 - 12.539459) & \text{if } 75 \le S_T < 85,\\
100(10 - 12.539459) & \text{if } 85 \le S_T.
\end{cases}
$$

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Example 8

(Use Table 1) Steve buys a 65*–strike call and a* 85*–strike put and sells a* 75*–strike call and a* 75*–strike put. All are for 100 shares and have expiration date one year from now. The current price of one share of XYZ stock is $75. The risk free annual effective rate of interest is* 5%*.*

(ii) Make a table with Steve's profit when the spot price at expiration is $60, $65, $70, $75, $80, $85, $90.

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Example 8

(Use Table 1) Steve buys a 65–strike call and a 85–strike put and sells a 75–strike call and a 75–strike put. All are for 100 shares and have expiration date one year from now. The current price of one share of XYZ stock is \$75. The risk free annual effective rate of interest is 5%.

(ii) Make a table with Steve's profit when the spot price at expiration is \$60, \$65, \$70, \$75, \$80, \$85, \$90.

Solution: (ii) table with Steve's profit is

Spot Price	60	65	70	75
Steve's profit	−253.95	−253.95	246.05	746.05

Spot Price	75	80	85	90
Steve's profit	746.05	246.05	−253.95	−253.95

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Example 8

(Use Table 1) Steve buys a 65–strike call and a 85–strike put and sells a 75–strike call and a 75–strike put. All are for 100 shares and have expiration date one year from now. The current price of one share of XYZ stock is $75. The risk free annual effective rate of interest is 5%.

(iii) Draw the graph of Steve's profit.

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Example 8

(Use Table 1) Steve buys a 65–strike call and a 85–strike put and sells a 75–strike call and a 75–strike put. All are for 100 shares and have expiration date one year from now. The current price of one share of XYZ stock is $75. The risk free annual effective rate of interest is 5%.

(iii) Draw the graph of Steve's profit.

Solution: (iii) The graph of the profit is Figure 10.

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Asymmetric Butterfly spread.

Given $0 < K_1 < K_2 < K_3$ and $0 < \lambda < 1$, a **butterfly spread** consists of: buying λ K_1–strike calls, buying $(1-\lambda)$ K_3–strike calls; and selling one K_2–strike call. The profit of this strategy is

$$
\begin{aligned}
&(\lambda)\max(S_T - K_1, 0) + (1-\lambda)\max(S_T - K_3, 0) - \max(S_T - K_2, 0) \\
&= \begin{cases} 0 - \mathrm{FVPrem} & \text{if } S_T < K_1, \\ \lambda(S_T - K_1) - \mathrm{FVPrem} & \text{if } K_1 \le S_T < K_2, \\ -\lambda K_1 + K_2 - (1-\lambda)S_T - \mathrm{FVPrem} & \text{if } K_2 \le S_T < K_3, \\ -\lambda K_1 + K_2 - (1-\lambda)K_3 - \mathrm{FVPrem} & \text{if } K_3 \le S_T, \end{cases}
\end{aligned}
$$

where

$$\mathrm{FVPrem} = (\lambda \mathrm{Call}(K_1, T) - \mathrm{Call}(K_2, T) + (1-\lambda)\mathrm{Call}(K_3, T))\,(1+i)^T.$$

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The profit of an symmetric butterfly spread is

$$
\begin{aligned}
&(\lambda)\max(S_T - K_1, 0) + (1-\lambda)\max(S_T - K_3, 0) - \max(S_T - K_2, 0) \\
=&\begin{cases} 0 - \mathrm{FVPrem} & \text{if } S_T < K_1, \\ \lambda(S_T - K_1) - \mathrm{FVPrem} & \text{if } K_1 \le S_T < K_2, \\ -\lambda K_1 + K_2 - (1-\lambda)S_T - \mathrm{FVPrem} & \text{if } K_2 \le S_T < K_3, \\ -\lambda K_1 + K_2 - (1-\lambda)K_3 - \mathrm{FVPrem} & \text{if } K_3 \le S_T, \end{cases}
\end{aligned}
$$

For an asymmetric butterfly spread, the profit functions increases in one interval and decreases in another interval. The rate of increase is not necessarily equal to the rate of decrease.

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Example 9

(Use Table 1) Karen buys seven 65*–strike calls, buys three* 85*–strike calls and sells ten* 75*–strike calls. Each option involves* 100 *shares of XYZ stock. Both have expiration date one year from now. The risk free annual effective rate of interest is* 5%*.*

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Example 9

(Use Table 1) Karen buys seven 65*–strike calls, buys three* 85*–strike calls and sells ten* 75*–strike calls. Each option involves 100 shares of XYZ stock. Both have expiration date one year from now. The risk free annual effective rate of interest is* 5%.

(i) Find Karen's profit as a function of the spot price at expiration.

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Example 9

(Use Table 1) Karen buys seven 65–*strike calls, buys three* 85–*strike calls and sells ten* 75–*strike calls. Each option involves* *100 shares of XYZ stock. Both have expiration date one year from now. The risk free annual effective rate of interest is* 5%.

(i) Find Karen's profit as a function of the spot price at expiration.

Solution: (i) The future value per share of the cost of entering the option contracts is

$$(7)(14.31722)(1.05) + (3)(7.78971)(1.05) - (3.680736)(1.05)$$
$$=35.0339184.$$

Karen's profit is

$$(100)\left((7)\max(S_T - 65, 0) + (3)\max(S_T - 85, 0)\right.$$
$$\left.-(10)\max(S_T - 75, 0) - (35.0339184)\right)$$

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Example 9

(Use Table 1) Karen buys seven 65–strike calls, buys three 85–strike calls and sells ten 75–strike calls. Each option involves 100 shares of XYZ stock. Both have expiration date one year from now. The risk free annual effective rate of interest is 5%.
(i) Find Karen's profit as a function of the spot price at expiration.
Solution: (i) (continuation)

$$(100)\,((7)\max(S_T-65,0)+(3)\max(S_T-85,0)$$
$$-(10)\max(S_T-75,0)-(35.0339184))$$
$$=\begin{cases} 100(-35.0339184) & \text{if } S_T<65,\\ 100((7)(S_T-65)-35.0339184) & \text{if } 65\le S_T<75,\\ 100((7)(S_T-65)-(10)(S_T-75) & \\ -35.0339184) & \text{if } 75\le S_T<85,\\ 100((7)(S_T-65)-(10)(S_T-75) & \\ +(3)(S_T-85)-35.0339184) & \text{if } 85\le S_T, \end{cases}$$

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Example 9

(Use Table 1) Karen buys seven 65*–strike calls, buys three* 85*–strike calls and sells ten* 75*–strike calls. Each option involves* 100 *shares of XYZ stock. Both have expiration date one year from now. The risk free annual effective rate of interest is* 5%*.*
(ii) Make a table with Karen's profit when the spot price at expiration is \$60, \$65, \$70, \$75, \$80, \$85, \$90.

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Example 9

(Use Table 1) Karen buys seven 65–strike calls, buys three 85–strike calls and sells ten 75–strike calls. Each option involves 100 shares of XYZ stock. Both have expiration date one year from now. The risk free annual effective rate of interest is 5%.

(ii) Make a table with Karen's profit when the spot price at expiration is \$60, \$65, \$70, \$75, \$80, \$85, \$90.

Solution: (ii) A table with Karen's profit is

Spot Price	60	65	70	75
Karen's profit	−3503.39	−3503.39	−3.39	3496.61

Spot Price	75	80	85	90
Karen's profit	3496.61	1996.61	496.61	496.61

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Example 9

(Use Table 1) Karen buys seven 65*–strike calls, buys three* 85*–strike calls and sells ten* 75*–strike calls. Each option involves* 100 *shares of XYZ stock. Both have expiration date one year from now. The risk free annual effective rate of interest is* 5%*.*
(iii) Draw the graph of Karen's profit.

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Example 9

(Use Table 1) Karen buys seven 65*–strike calls, buys three* 85*–strike calls and sells ten* 75*–strike calls. Each option involves 100 shares of XYZ stock. Both have expiration date one year from now. The risk free annual effective rate of interest is* 5%*.*
(iii) Draw the graph of Karen's profit.
Solution: (iii) The graph of the profit is on Figure 11.

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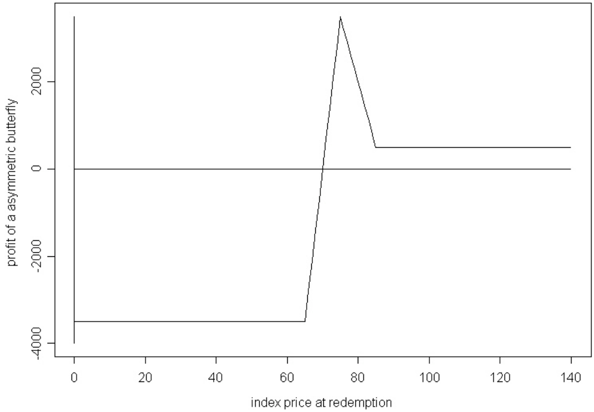

Figure 11: Profit for an asymmetric butterfly.

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Box spreads.

A **box spread** is a combination of options which create a synthetic long forward at one price and a synthetic short forward at a different price. Let K_1 be the price of the synthetic long forward. Let K_2 be the price of the synthetic short forward. With a box spread, you are able to buy an asset for K_1 at time T and sell it for K_2 at time T not matter the spot price at expiration. At time T, a payment of $K_2 - K_1$ per share is obtained. A box spread can be obtained from:
(i) buy a K_1–strike call and sell a K_1–strike put.
(ii) sell a K_2–strike call and buy a K_2–strike put.
If put–call parity holds, the premium per share to enter these option contract is

$$\begin{aligned}&\mathrm{Call}(K_1, T) - \mathrm{Put}(K_1, T) - (\mathrm{Call}(K_2, T) - \mathrm{Put}(K_2, T))\\ =&(K_2 - K_1)(1 + i)^{-T}.\end{aligned}$$

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- If $K_1 < K_2$, a box spread is a way to lend money. An investment of $(K_2 - K_1)(1+i)^{-T}$ per share is made at time zero and a return of $K_2 - K_1$ per share is obtained at time T.
- If $K_1 > K_2$, a box spread is a way to borrow money. A return of $(K_1 - K_2)(1+i)^{-T}$ per share is received at time zero and a loan payment of $K_1 - K_2$ per share is made at time T.

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Example 10

Mario needs \$50,000 to open a pizzeria. He can borrow at the annual effective rate of interest of 8.5%. Mario also can buy/sell three–year options on XYZ stock with the following premiums per share:

$\mathrm{Call}(K, T)$	14.42	7.78
$\mathrm{Put}(K, T)$	7.37	17.29
K	70	90

Mario buys a 90–strike call and a 70–strike put, and sells a 90–strike put and a 70–strike call. All the options are for the same nominal amount. Mario receives a total of \$50000 from these sales. Find Mario's cost at expiration time to settle these options. Find the annual rate of return that Mario gets on this "loan".

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Solution: The price per share of Mario's portfolio is $7.78 + 7.37 - 17.29 - 14.42 = -16.56$. So, the nominal amount of each option is $\frac{50000}{16.56} = 3019.3237$. Initially, Mario gets \$50000 for entering these option contracts. In three years, Mario buys at \$90 per share and sells at \$70 per share. Hence, Mario pays $(3019.3237)(90 - 70) = 60386.474$ for settling the option contracts. Let i be the annual effective rate of interest on this loan. We have that $50000(1 + i)^3 = 60386.474$ and $i = 6.493529844\%$.

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