Manual for SOA Exam MLC.

Chapter 2. Survival models.
Actuarial problems.

Extract from:
"Arcones' Manual for SOA Exam MLC. Fall 2009 Edition",
available at http://www.actexmadriver.com/

(#21, Exam M, Spring 2005) You are given:

(i) $\overset{\circ}{e}_{30:\overline{40}|} = 27.692$

(ii) $s(x) = 1 - \frac{x}{\omega}$, $0 \le x \le \omega$.

(iii) $T(x)$ is the future lifetime random variable for (x).

Calculate $\mathrm{Var}(T(30))$.

(A) 332 (B) 352 (C) 372 (D) 392 (E) 412

(#33, Exam M, Spring 2005) You are given:

$$\mu(x) = \begin{cases} 0.05 & 50 \leq x < 60 \\ 0.04 & 60 \leq x < 70 \end{cases}$$

Calculate ${}_{4|14}q_{50}$.
(A) 0.38 (B) 0.39 (C) 0.41 (D) 0.43 (E) 0.44

(#31, Exam M, Fall 2005) The graph of a piecewise linear survival function, $s(x)$, consists of 3 line segments with endpoints (0, 1), (25, 0.50), (75, 0.40), (100, 0).
Calculate $\frac{{}_{20|55}q_{15}}{{}_{55}q_{35}}$.
(A) 0.69 (B) 0.71 (C) 0.73 (D) 0.75 (E) 0.77

(#32, Exam M, Fall 2005) For a group of lives aged 30, containing an equal number of smokers and non–smokers, you are given:
(i) For non-smokers, $\mu^n(x) = 0.08$, $x \geq 30$.
(ii) For smokers, $\mu^s(x) = 0.16$, $x \geq 30$.
Calculate q_{80} for a life randomly selected from those surviving to age 80.
(A) 0.078 (B) 0.086 (C) 0.095 (D) 0.104 (E) 0.112

(#35, Exam M, Fall 2005) An actuary for a medical device manufacturer initially models the failure time for a particular device with an exponential distribution with mean 4 years. This distribution is replaced with a spliced model whose density function:

(i) is uniform over $[0, 3]$

(ii) is proportional to the initial modeled density function after 3 years

(iii) is continuous

Calculate the probability of failure in the first 3 years under the revised distribution.

(A) 0.43 (B) 0.45 (C) 0.47 (D) 0.49 (E) 0.51

(#2, Exam M, Fall 2006) You are given the survival function

$$s(x) = 1 - (0.01x)^2, 0 \leq x \leq 100.$$

Calculate $\overset{\circ}{e}_{30:\overline{50}|}$, the 50-year temporary complete expectation of life of (30).

(A) 27 (B) 30 (C) 34 (D) 37 (E) 41

(#13, Exam M, Fall 2005) The actuarial department for the SharpPoint Corporation models the lifetime of pencil sharpeners from purchase using a generalized De Moivre model with $s(x) = (1 - x/\omega)^{\alpha}$, for $\alpha > 0$ and $0 \le x \le \omega$.
A senior actuary examining mortality tables for pencil sharpeners has determined that the original value of α must change. You are given:
(i) The new complete expectation of life at purchase is half what it was previously.
(ii) The new force of mortality for pencil sharpeners is 2.25 times the previous force of mortality for all durations.
(iii) ω remains the same.
Calculate the original value of α.
(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

(#14, Exam M, Fall 2006) You are given:
(i) T is the future lifetime random variable.
(ii) $\mu(t) = \mu$, $t \geq 0$
(iii) $\mathrm{Var}[T] = 100$.
Calculate $E[T \wedge 10]$.
(A) 2.6 (B) 5.4 (C) 6.3 (D) 9.5 (E) 10.0

(#16, Exam M, Fall 2006) You are given the following information on participants entering a special 2-year program for treatment of a disease:

(i) Only 10% survive to the end of the second year.

(ii) The force of mortality is constant within each year.

(iii) The force of mortality for year 2 is three times the force of mortality for year 1.

Calculate the probability that a participant who survives to the end of month 3 dies by the end of month 21.

(A) 0.61 (B) 0.66 (C) 0.71 (D) 0.75 (E) 0.82

(#23, Exam M, Fall 2006) You are given 3 mortality assumptions:
(i) Illustrative Life Table (ILT),
(ii) Constant force model (CF), where $s(x) = e^{-\mu x}$ $x > 0$,
(iii) DeMoivre model (DM), where $s(x) = 1 - \frac{x}{\omega}$, $0 \le x \le \omega$,
For the constant force and DeMoivre models, ${}_2p_{70}$ is the same as for the Illustrative Life Table.
Rank $e_{70:\overline{2}|}$ for these 3 models.
(A) $ILT < CF < DM$ (B) $ILT < DM < CF$ (C) $CF < DM < ILT$ (D) $DM < CF < ILT$ (E) $DM < ILT < CF$

(#1, Exam MLC, Spring 2007) You are given:
(i) ${}_3p_{70} = 0.95$
(ii) ${}_2p_{71} = 0.96$
(iii) $\int_{71}^{75} \mu_x \, dx = 0.107$
Calculate ${}_5p_{70}$.
(A) 0.85 (B) 0.86 (C) 0.87 (D) 0.88 (E) 0.89

(#13, MLC–09–08) A population has 30% who are smokers with a constant force of mortality 0.2 and 70% who are non-smokers with a constant force of mortality 0.1. Calculate the 75^{th} percentile of the distribution of the future lifetime of an individual selected at random from this population.

(A) 10.7 (B) 11.0 (C) 11.2 (D) 11.6 (E) 11.8

(#21, MLC–09–08) For (x):
(i) K is the curtate future lifetime random variable.
(ii) $q_{x+k} = 0.1(k+1)$, $k = 0, 1, 2, \dots, 9$
(iii) $X = \min(K, 3)$
Calculate $\mathrm{Var}(X)$.
(A) 1.1 (B) 1.2 (C) 1.3 (D) 1.4 (E) 1.5

(#22, MLC–09–08) For a population which contains equal numbers of males and females at birth:
(i) For males, $\mu^{(m)}(x) = 0.10$, $x \geq 0$
(ii) For females, $\mu^{(f)}(x) = 0.08$, $x \geq 0$
Calculate q_{60} for this population.
(A) 0.076 (B) 0.081 (C) 0.086 (D) 0.091 (E) 0.096

(#28, MLC–09–08) For T, the future lifetime random variable for (0):
(i) $\omega > 70$
(ii) ${}_{40}p_0 = 0.6$
(iii) $E(T) = 62$
(iv) $E[\min(T, t)] = t - 0.005t^2$, $0 < t < 60$
Calculate the complete expectation of life at 40.
(A) 30 (B) 35 (C) 40 (D) 45 (E) 50

(#32, MLC–09–08) Given: The survival function $s(x)$, where
$s(x) = 1$, $0 \leq x < 1$
$s(x) = 1 - \left\{ \left(e^x \right) /100 \right\}$, $1 \leq x < 4.5$
$s(x) = 0$, $4.5 < x$
Calculate $\mu(4)$.
(A) 0.45 (B) 0.55 (C) 0.80 (D) 1.00 (E) 1.20

(#59, MLC–09–08) You are given:

(i) $R = 1 - e^{-\int_0^1 \mu_{x+t}\, dt}$

(ii) $S = 1 - e^{-\int_0^1 (\mu_{x+t}+k)\, dt}$

(iii) k is a constant such that $S = 0.75R$

Determine an expression for k.

(A) $\ln((1 - q_x)/(1 - 075q_x))$ (B) $\ln((1 - 0.75q_x)/(1 - p_x))$

(C) $\ln((1 - 0.75p_x)/(1 - p_x))$ (D) $\ln((1 - p_x)/(1 - 0.75q_x))$

(E) $\ln((1 - 0.75q_x)/(1 - q_x))$

(#65, MLC–09–08) You are given:

$$\mu(x) = \begin{cases} 0.04 & \text{if } 0 < x < 40 \\ 0.05 & \text{if } x > 40 \end{cases}$$

Calculate $\overset{\circ}{e}_{25:\overline{25}|}$.
(A) 14.0 (B) 14.4 (C) 14.8 (D) 15.2 (E) 15.6

(#98, MLC–09–08) For a given life age 30, it is estimated that an impact of a medical breakthrough will be an increase of 4 years in $\overset{\circ}{e}_{30}$, the complete expectation of life.

Prior to the medical breakthrough, $s(x)$ followed de Moivre's law with $\omega = 100$ as the limiting age.

Assuming de Moivre's law still applies after the medical breakthrough, calculate the new limiting age.

(A) 104 (B) 105 (C) 106 (D) 107 (E) 108

(#116, MLC–09–08) For a population of individuals, you are given:
(i) Each individual has a constant force of mortality.
(ii) The forces of mortality are uniformly distributed over the interval (0,2).
Calculate the probability that an individual drawn at random from this population dies within one year.
(A) 0.37 (B) 0.43 (C) 0.50 (D) 0.57 (E) 0.63

(#120, MLC–09–08) For a 4-year college, you are given the following probabilities for dropout from all causes:

$$q_0 = 0.15 \quad q_1 = 0.10 \quad q_2 = 0.05 \quad q_3 = 0.01.$$

Dropouts are uniformly distributed over each year. Compute the temporary 1.5-year complete expected college lifetime of a student entering the second year, $\mathring{e}_{1:\overline{1.5}|}$.

(A) 1.25 (B) 1.30 (C) 1.35 (D) 1.40 (E) 1.45

(#131, MLC–09–08) Mortality for Audra, age 25, follows De Moivre's law with $\omega = 100$. If she takes up hot air ballooning for the coming year, her assumed mortality will be adjusted so that for the coming year only, she will have a constant force of mortality of 0.1.

Calculate the decrease in the 11-year temporary complete life expectancy for Audra if she takes up hot air ballooning.

(A) 0.10 (B) 0.35 (C) 0.60 (D) 0.80 (E) 1.00

(#145, MLC–09–08) Given:
(i) Superscripts M and N identify two forces of mortality and the curtate expectations of life calculated from them.
(ii) $\mu_{25}^{N}(t) = \begin{cases} \mu_{25}^{M}(t) + (0.1)*(1-t) & 0 \le t \le 1 \\ \mu_{25}^{M}(t) & t > 1 \end{cases}$
(iii) $e_{25}^{M} = 10.0$
Calculate e_{25}^{N}.
(A) 9.2 (B) 9.3 (C) 9.4 (D) 9.5 (E) 9.6

(#155, MLC–09–08) Given:
(i) $\mu(x) = F + e^{2x}$, $x \geq 0$
(ii) ${}_{0.4}p_0 = 0.50$
Calculate F.
(A) -0.20 (B) -0.09 (C) 0.00 (D) 0.09 (E) 0.20

(#161, MLC–09–08) You are given:
(i) $\overset{\circ}{e}_{30:\overline{40}|} = 27.692$
(ii) $s(x) = 1 - \frac{x}{\omega}, \quad 0 \le x \le \omega$
(iii) $T(x)$ is the future lifetime random variable for (x).
Calculate $\mathrm{Var}(T(30))$.
(A) 332 (B) 352 (C) 372 (D) 392 (E) 412

(#189, MLC–09–08) You are given:
(i) T is the future lifetime random variable.
(ii) $\mu(t) = \mu$, $t \geq 0$
(iii) $\mathrm{Var}[T] = 100$.
(iv) $X = T \wedge 10$
Calculate $E[X]$.
(A) 2.6 (B) 5.4 (C) 6.3 (D) 9.5 (E) 10.0

