Manual for SOA Exam MLC.

Chapter 10. Poisson processes.

Section 10.6. Compound Poisson process.

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Compound Poisson processes

Definition 1

A stochastic process $\{X(t) : t \geq 0\}$ *is said to be a* **compound Poisson process** *if it can be represented as*

$$X(t) = \begin{cases} \sum_{i=1}^{N(t)} Y_i & \text{if } N(t) \geq 1, \\ 0 & \text{if } N(t) = 0, \end{cases}$$

where $\{N(t) : t \geq 0\}$ *is a Poisson process and* $\{Y_i\}_{i=1}^{\infty}$ *is a sequence of i.i.d.r.v.'s independent of* $\{N(t) : t \geq 0\}$.

Theorem 1

For a compound Poisson process $\{X(t) : t \geq 0\}$,

$$E[X(t)] = \lambda t E[Y_1] \text{ and } \mathrm{Var}(X(t)) = \lambda t E[Y_1^2].$$

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Proof: Using the double expectation theorem, we have that

$$E[X(t)|N(t)=n] = E\left[\sum_{i=1}^{N(t)} Y_i | N(t) = n\right]$$

$$= E\left[\sum_{i=1}^{n} Y_i | N(t) = n\right] = nE[Y_1],$$

$$E[X(t)] = E[E[X(t)|N(t)=n]] = E[N(t)E[Y_1]] = \lambda t E[Y_1].$$

Theorem 1

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$$E[X(t)] = \lambda t E[Y_1] \text{ and } \mathrm{Var}(X(t)) = \lambda t E[Y_1^2].$$

Proof: Using the double expectation theorem, we have that

$$\mathrm{Var}(X(t)|N(t) = n) = \mathrm{Var}(\sum_{i=1}^{N(t)} Y_i | N(t) = n)$$

$$= \mathrm{Var}(\sum_{i=1}^{n} Y_i | N(t) = n) = n\mathrm{Var}(Y_1),$$

$$\mathrm{Var}(X(t)) = E[\mathrm{Var}(X(t)|N(t) = n)] + \mathrm{Var}(E[X(t)|N(t) = n])$$

$$= E[N(t)\mathrm{Var}(Y_1)] + \mathrm{Var}(N(t)E[Y_1]) = \lambda t\mathrm{Var}(Y_1) + \lambda t(E[Y_1])^2$$

$$= \lambda t E[Y_1^2].$$

A possible model for the amount of total claims which an insurance company receives follows. Let $N(t)$ be the number of claims that the company receives until time t. Assume that $\{N(t) : t \geq 0\}$ follows a Poisson process with rate λ. Let $\{Y_j\}$ be a sequence of claims which the insurance company gets. Assume that $\{Y_j\}$ is a sequence of i.i.d.r.v.'s. Suppose that $\{N(t) : t \geq 0\}$ and $\{Y_j\}$ are independent. Let $X(t) = \sum_{i=1}^{N(t)} Y_i$ be the amount of claims received until time t. $X(t)$ is called the **aggregate claims**.

Example 1

The number of dental claims received by an insurance company follows a Poisson process with rate $\lambda = 50$ *claims/day. The claim amounts are independent and uniformly distributed over* $[0, 300]$. *Find the mean and the standard deviation of the total claim amounts received in a 30 days period.*

Example 1

The number of dental claims received by an insurance company follows a Poisson process with rate $\lambda = 50$ claims/day. The claim amounts are independent and uniformly distributed over $[0, 300]$. Find the mean and the standard deviation of the total claim amounts received in a 30 days period.

Solution: We have that

$$E[X(30)] = (30)\lambda E[Y_1] = (30)(50)(150) = 225000$$

and

$$\mathrm{Var}(X(30)) = (30)\lambda E[X^2] = (30)(50)\frac{300^2}{3} = 45000000.$$

