Manual for SOA Exam MLC.

Chapter 3. Life tables.
Section 3.6. Select and ultimate tables.

Extract from:
"Arcones' Manual for SOA Exam MLC. Fall 2009 Edition",
available at http://www.actexmadriver.com/

Select and ultimate tables

A **select table** is a mortality table for a group of people subject to a special circumstance (disability, retirement, etc.). Usually, the cohort of people is given by a certain age. Suppose that we start with $\ell_{[x]}$ lives of a certain cohort at time x. The number of survivors at time t is denoted by $\ell_{[x]+t}$. We denote by

$$_np_{[x]+t} = \frac{\ell_{[x]+t+n}}{\ell_{[x]+t}} \text{ and } {}_nq_{[x]+t} = \frac{\ell_{[x]+t} - \ell_{[x]+t+n}}{\ell_{[x]+t}}.$$

When $n = 1$, we use the notation

$$p_{[x]+t} = \frac{\ell_{[x]+t+1}}{\ell_{[x]+t}} \text{ and } q_{[x]+t} = \frac{\ell_{[x]+t} - \ell_{[x]+t-1}}{\ell_{[x]+t}}.$$

Notice that

$$p_{[x]}p_{[x]+1}\cdots p_{[x]+n-1} = \frac{\ell_{[x]+1}}{\ell_{[x]}}\frac{\ell_{[x]+2}}{\ell_{[x]+1}}\dots,\frac{\ell_{[x]+n}}{\ell_{[x]+n-1}} = \frac{\ell_{[x]+n}}{\ell_{[x]}} = {}_np_{[x]}.$$

Example 1

Consider the following select table:

x	$q_{[x]}$	$q_{[x]+1}$	$q_{[x]+2}$
35	0.013	0.012	0.011

If $\ell_{[35]} = 1000$, *find* $\ell_{[35]+1}$, $\ell_{[35]+2}$ *and* $\ell_{[35]+3}$.

Example 1

Consider the following select table:

x	$q_{[x]}$	$q_{[x]+1}$	$q_{[x]+2}$
35	*0.013*	*0.012*	*0.011*

If $\ell_{[35]} = 1000$, *find* $\ell_{[35]+1}$, $\ell_{[35]+2}$ *and* $\ell_{[35]+3}$.

Solution:

From $p_{[x]+t} = \frac{\ell_{[x]+t+1}}{\ell_{[x]+t}}$, we get $\ell_{[x]+t+1} = \ell_{[x]+t}p_{[x]+t}$. Hence,

$$\ell_{[35]+1} = \ell_{[35]}p_{[35]} = (1000)(1-0.013) = 987,$$
$$\ell_{[35]+2} = \ell_{[35]+1}p_{[35]+1} = (987)(1-0.012) = 975.156,$$
$$\ell_{[35]+3} = \ell_{[35]+2}p_{[35]+2} = (975.156)(1-0.011) = 964.429284.$$

We get

x	$\ell_{[x]}$	$\ell_{[x]+1}$	$\ell_{[x]+2}$	$\ell_{[x]+3}$
35	1000	987	975.156	964.429284

A **select and ultimate table** displays the number of living using a select table for a certain number of years and a standard life table when the elapsed time is bigger than this number of years. The number of years such that the select table is used is called the **select period**. A life table which does not use the select period is called an **ultimate table**.
Suppose that select period is three years. Then, a select and ultimate life table has the form

$[x]$	$\ell_{[x]}$	$\ell_{[x]+1}$	$\ell_{[x]+2}$	ℓ_{x+3}	$x+3$
$[1]$	$\ell_{[1]}$	$\ell_{[1]+1}$	$\ell_{[1]+2}$	ℓ_4	4
$[2]$	$\ell_{[2]}$	$\ell_{[2]+1}$	$\ell_{[2]+2}$	ℓ_5	5
$[3]$	$\ell_{[3]}$	$\ell_{[3]+1}$	$\ell_{[3]+2}$	ℓ_6	6

Example 2

You are given the following extract from a 2–year select–and–ultimate mortality table:

$[x]$	$\ell_{[x]}$	$\ell_{[x]+1}$	ℓ_{x+2}	$x+2$
45	1235	1124	1039	47
46	1135	1025	978	48
47	1012	996	965	49

(i) Complete the table

$[x]$	$q_{[x]}$	$q_{[x]+1}$	q_{x+2}	$x+2$
45				47
46				48
47			—	49

(ii) Find ${}_2p_{[47]}$, ${}_2p_{[46]+1}$ and ${}_2p_{47}$.

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$[x]$	$\ell_{[x]}$	$\ell_{[x]+1}$	ℓ_{x+2}	$x+2$
45	1235	1124	1039	47
46	1135	1025	978	48
47	1012	996	965	49

Solution: (i)

$[x]$	$q_{[x]}$	$q_{[x]+1}$	q_{x+2}	$x+2$
45	$\frac{1235-1124}{1235}$	$\frac{1124-1039}{1124}$	$\frac{1039-978}{1039}$	47
46	$\frac{1135-1025}{1135}$	$\frac{1025-978}{1025}$	$\frac{978-965}{978}$	48
47	$\frac{1012-996}{1012}$	$\frac{996-965}{996}$	$-$	49

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45	1235	1124	1039	47
46	1135	1025	978	48
47	1012	996	965	49

(ii) Find ${}_2p_{[47]}$, ${}_2p_{[46]+1}$ and ${}_2p_{47}$.

Solution: (ii) We have that

$$ {}_2p_{[47]} = \frac{\ell_{49}}{\ell_{[47]}} = \frac{965}{1012} = 0.9535573123 $$

$$ {}_2p_{[46]+1} = \frac{\ell_{49}}{\ell_{[46]+1}} = \frac{965}{1025} = 0.9414634146 $$

$$ {}_2p_{47} = \frac{\ell_{49}}{\ell_{47}} = \frac{965}{1039} = 0.9287776708 $$

Example 3

You are given the following extract from a 2–year select–and–ultimate mortality table:

$[x]$	$\ell_{[x]}$	$\ell_{[x]+1}$	ℓ_{x+2}	$x+2$
45	1235	1124	1039	47
46	1135	1025	978	48
47	1012	996	965	49

(i) Complete the table

$[x]$	$q_{[x]}$	$q_{[x]+1}$	q_{x+2}	$x+2$
45				47
46				48
47			–	49

(ii) Find ${}_2p_{[47]}$, ${}_2p_{[46]+1}$ and ${}_2p_{47}$.

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Solution: (i)

$[x]$	$q_{[x]}$	$q_{[x]+1}$	q_{x+2}	$x+2$
45	$\frac{1235-1124}{1235}$	$\frac{1124-1039}{1124}$	$\frac{1039-978}{1039}$	47
46	$\frac{1135-1025}{1135}$	$\frac{1025-978}{1025}$	$\frac{978-965}{978}$	48
47	$\frac{1012-996}{1012}$	$\frac{996-965}{996}$	$-$	49

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(ii) Find ${}_2p_{[47]}$, ${}_2p_{[46]+1}$ and ${}_2p_{47}$.

Solution: (ii) We have that

$$ {}_2p_{[47]} = \frac{\ell_{49}}{\ell_{[47]}} = \frac{965}{1012} = 0.9535573123 $$

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$$ {}_2p_{47} = \frac{\ell_{49}}{\ell_{47}} = \frac{965}{1039} = 0.9287776708 $$

Example 4

You are given the following extract from a 2–year select–and–ultimate mortality table:

$[x]$	$q_{[x]}$	$q_{[x]+1}$	q_{x+2}	$x+2$
45	0.009	0.008	0.007	47
46	0.008	0.006	0.005	48

Complete the table

$[x]$	$\ell_{[x]}$	$\ell_{[x]+1}$	ℓ_{x+2}	$x+2$
45	10000			47
46				48

Example 4

You are given the following extract from a 2–year select–and–ultimate mortality table:

$[x]$	$q_{[x]}$	$q_{[x]+1}$	q_{x+2}	$x+2$
45	0.009	0.008	0.007	47
46	0.008	0.006	0.005	48

$$\ell_{[45]+1} = \ell_{[45]} p_{[45]} = (10000)(1-0.009) = 9910,$$

$$\ell_{47} = \ell_{[45]+1} p_{[45]+1} = (10000)(1-0.009)(1-0.008) = 9830.72,$$

$$\ell_{48} = \ell_{47} p_{47} = (10000)(1-0.009)(1-0.008)(1-0.007) = 9761.90496,$$

$$\ell_{[46]+1} = \frac{\ell_{[48]}}{p_{[46]+1}} = \frac{(10000)(1-0.009)(1-0.008)(1-0.007)}{(1-0.006)} = 9820.82$$

$$\ell_{[46]} = \frac{\ell_{[46]+1}}{p_{[46]}} = \frac{(10000)(1-0.009)(1-0.008)(1-0.007)}{((1-0.006)(1-0.008)} = 9900.0301$$

Example 4

You are given the following extract from a 2–year select–and–ultimate mortality table:

$[x]$	$q_{[x]}$	$q_{[x]+1}$	q_{x+2}	$x+2$
45	0.009	0.008	0.007	47
46	0.008	0.006	0.005	48

$[x]$	$\ell_{[x]}$	$\ell_{[x]+1}$	ℓ_{x+2}	$x+2$
45	10000	9910	9830.72	47
46	9900.030181	9820.82994	9761.90496	48

