Manual for SOA Exam MLC.

Chapter 4. Life Insurance.

Section 4.4. Non-level payments paid at the end of the year.

Extract from:
"Arcones' Manual for the SOA Exam MLC. Fall 2009 Edition".
available at http://www.actexmadriver.com/

Non-level payments paid at the end of the year

Suppose that a life insurance provides a benefit of b_k paid at the end of the k–th year if death happens in this year. The present value of this benefit is

$$B_x = b_K \nu^K.$$

The actuarial present value of B_x is

$$E[B_x] = \sum_{k=1}^{\infty} b_k \nu^k \mathbb{P}\{K_x = k\} = \sum_{k=1}^{\infty} b_k \nu^k \cdot {}_{k-1|}q_x.$$

We have that

$$E[B_x^2] = \sum_{k=1}^{\infty} b_k^2 \nu^{2k} \mathbb{P}\{K_x = k\} = \sum_{k=1}^{\infty} b_k^2 \nu^{2k} \cdot {}_{k-1|}q_x.$$

Example 1

A whole life insurance on (50) *pays 50000 plus the return of the net single premium with interest at* $\delta = 0.03$ *at the end of the year of time of death. The survival function for (50) follows the de Moivre's law with* $\omega = 110$. *Calculate the net single premium for* $\delta = 0.07$.

Example 1

A whole life insurance on (50) *pays 50000 plus the return of the net single premium with interest at* $\delta = 0.03$ *at the end of the year of time of death. The survival function for (50) follows the de Moivre's law with* $\omega = 110$*. Calculate the net single premium for* $\delta = 0.07$*.*

Solution: We have that $b_k = 50000 + Pe^{(0.03)k}$ and

$$P = \sum_{k=1}^{\infty} b_k \nu^k \cdot {}_{k-1|}q_x = \sum_{k=1}^{60} (50000 + Pe^{(0.03)k}) e^{-(0.07)k} \frac{1}{60}$$

$$= \frac{50000}{60} \sum_{k=1}^{60} e^{-(0.07)k} + \frac{P}{60} \sum_{k=1}^{60} e^{-(0.04)k}$$

$$= \frac{(50000)(e^{-0.07} - e^{-(0.07)61})}{60(1 - e^{-0.07})} + \frac{(e^{-0.04} - e^{-(0.04)61})P}{60(1 - e^{-0.04})}$$

$$= 11320.61245 + 0.3713406834P.$$

Hence, $P = \frac{11320.61245}{1 - 0.3713406834} = 18007.54741$.

Definition 1

An **increasing by one whole life insurance** *pays k at time k, for each $k \geq 1$, if the failure happens in the k–th interval.*

Definition 2

The actuarial present value of a unit increasing whole life insurance is denoted by $(IA)_x$.

We have that

$$(IA)_x = \sum_{k=1}^{\infty} k\nu^k \mathbb{P}\{K_x = k\} = \sum_{k=1}^{\infty} k\nu^k \cdot {}_{k-1}|q_x.$$

Theorem 1

For each $x \geq 0$,

$$(IA)_x = A_x + {}_1E_x \, (IA)_{x+1} \, .$$

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Proof: We have that

$$_1E_x\,(IA)_{x+1} = \nu\frac{s(x+1)}{s(x)}\sum_{k=1}^{\infty} k\nu^k\frac{s(x+1+k-1)-s(x+1+k)}{s(x+1)}$$

$$=\sum_{k=1}^{\infty} k\nu^{k+1}\frac{s(x+1+k-1)-s(x+1+k)}{s(x)}$$

$$=\sum_{k=2}^{\infty}(k-1)\nu^k\frac{s(x+k-1)-s(x+k)}{s(x)},$$

Theorem 1

For each $x \geq 0$,

$$(IA)_x = A_x + {}_1E_x\,(IA)_{x+1}\,.$$

Proof:

$$\begin{aligned}
&A_x + {}_1E_x\,(IA)_{x+1}\\
=&\sum_{k=1}^{\infty}\nu^k\frac{s(x+k-1)-s(x+k)}{s(x)}\\
&+\sum_{k=2}^{\infty}(k-1)\nu^k\frac{s(x+k-1)-s(x+k)}{s(x)}\\
=&\sum_{k=1}^{\infty}k\nu^k\frac{s(x+k-1)-s(x+k)}{s(x)} = (IA)_x\,.
\end{aligned}$$

Example 2

Suppose that $A_{30} = 0.13$, $(IA)_{30} = 0.45$, $\nu = 0.94$ *and* $p_{30} = 0.99$. *Find* $(IA)_{31}$.

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Solution: Since

$$(IA)_x = A_x + {}_1E_x \, (IA)_{x+1} \,,$$

$$0.45 = 0.13 + (0.94)(0.99)\,(IA)_{31} \,,$$

and $(IA)_{31} = \frac{0.45-0.13}{(0.94)(0.99)} = 0.3438641737.$

Definition 3

An **increasing by one n–th year term life insurance** *pays k at time k, where $k \geq 1$, if the failure happens in the k—th interval and $k \leq n$.*

Definition 4

The actuarial present value of a unit increasing n–th year term life insurance is denoted by $(IA)^1_{x:\overline{n}|}$.

We have that

$$(IA)^1_{x:\overline{n}|} = \sum_{k=1}^{n} k\nu^k \mathbb{P}\{K_x = k\} = \sum_{k=1}^{n} k\nu^k \cdot {}_{k-1}|q_x.$$

Definition 5

An **decreasing by one n–th year term life insurance** *pays $n+1-k$ at time k if the failure happens in the k—th interval, where $1 \leq k \leq n$.*

Definition 6

The actuarial present value of a unit decreasing n–th year term life insurance is denoted by $(DA)^1_{x:\overline{n}|}$.

We have that

$$(DA)^1_{x:\overline{n}|} = \sum_{k=1}^{n} (n+1-k)\nu^k \mathbb{P}\{K_x = k\} = \sum_{k=1}^{n} (n+1-k)\nu^k \cdot {}_{k-1|}q_x.$$

Theorem 2

$$(IA)_{x:\overline{n}|} + (DA)_{x:\overline{n}|} = (n+1)\,(A)_{x:\overline{n}|}\,.$$

Proof:

$$(IA)^{1}_{x:\overline{n}|} + (DA)^{1}_{x:\overline{n}|} = \sum_{k=1}^{n} k\nu^k \cdot {}_{k-1}|q_x + \sum_{k=1}^{n}(n+1-k)\nu^k \cdot {}_{k-1}|q_x$$

$$= \sum_{k=1}^{n}(n+1)\nu^k \cdot {}_{k-1}|q_x = (n+1)\,(A)^{1}_{x:\overline{n}|}\,.$$

