Manual for SOA Exam MLC.

Chapter 4. Life Insurance.

Section 4.5. Payments at the end of the m–thly time interval.

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Payments at the end of the m–thly time interval

It is unusual that claims are paid at the end of the year. A better model uses that claims are paid at the end of each month, or other period. In section, we consider the case when payments can be made at m different equally spaced times a year. Previous insurance quantities are defined as before. To indicate that payments are made m–thly, a superindex $^{(m)}$ is added to the actuarial notation of insurance variables.

Suppose that a whole life insurance is paid at the end of the m–thly time interval in which failure occurs. Let $J_x^{(m)} = j$ if $T_x \in \left(\frac{j-1}{m}, \frac{j}{m}\right]$, for some integer $j \geq 1$. The present value of a whole life insurance paid the end of the m–thly time interval in which failure occurs is

$$Z_x^{(m)} = \nu^{J_x^{(m)}/m}$$

Hence,

$$A_x^{(m)} = E[Z_x^{(m)}] = \sum_{j=1}^{\infty} \nu^{j/m} \mathbb{P}\{J_x^{(m)} = j\},$$

$${}^2A_x^{(m)} = E[(Z_x^{(m)})^2] = \sum_{j=1}^{\infty} \nu^{2j/m} \mathbb{P}\{J_x^{(m)} = j\},$$

$$\mathrm{Var}(Z_x^{(m)}] = {}^2A_x^{(m)} - (A_x^{(m)})^2.$$

Suppose that an n–year term life insurance guarantees a payment at the end of the m–thly time interval in which failure occurs. The present value of this life insurance is

$$\overset{(m)}{Z}{}^{1}_{x:\overline{1}|} = v^{J_x^{(m)}/m} I(J_x^{(m)} \le nm) = \begin{cases} v^{J_x^{(m)}/m} & \text{if } J_x^{(m)} \le nm, \\ 0 & \text{if } J_x^{(m)} > nm. \end{cases}$$

In this case, we have that

$$\overset{(m)}{A}{}^{1}_{x:\overline{1}|} = E[\overset{(m)}{Z}{}^{1}_{x:\overline{1}|}] = \sum_{j=1}^{nm} \nu^{j/m} \mathbb{P}\{J_x^{(m)} = j\},$$

$$^{2}A^{(m)}_{x:\overline{n}|} = E[(\overset{(m)}{Z}{}^{1}_{x:\overline{1}|})^2] = \sum_{j=1}^{nm} \nu^{2j/m} \mathbb{P}\{J_x^{(m)} = j\},$$

$$\mathrm{Var}(\overset{(m)}{Z}{}^{1}_{x:\overline{1}|}) = {}^{2}\overset{(m)}{A}{}^{1}_{x:\overline{1}|} - (\overset{(m)}{A}{}^{1}_{x:\overline{1}|})^2.$$

Suppose that an n–year deferred life insurance guarantees a payment at the end of the m–thly time interval in which failure occurs (if this death happens after n years). The present value of this life insurance is

$$ {}_{n|}Z_x^{(m)} = \nu^{J_x^{(m)}/m} I(J_x^{(m)} > nm) = \begin{cases} \nu^{J_x^{(m)}/m} & \text{if } J_x^{(m)} > nm, \\ 0 & \text{if } J_x^{(m)} \le nm, \end{cases} $$

In this case, we have that

$$ {}_{n|}A_x^{(m)} = E[Z_{x:\overline{n}|}^{(m)}] = \sum_{j=nm+1}^{\infty} \nu^{j/m} \mathbb{P}\{J_x^{(m)} = j\}, $$

$$ {}^2{}_{n|}A_x^{(m)} = E[(Z_{x:\overline{n}|}^{(m)})^2] = \sum_{j=nm+1}^{\infty} \nu^{2j/m} \mathbb{P}\{J_x^{(m)} = j\}, $$

$$ \mathrm{Var}({}_{n|}Z_x^{(m)}) = {}^2{}_{n|}A_x^{(m)} - ({}_{n|}A_x^{(m)})^2. $$

Example 1

Rose is 40 years old. She buys a whole life policy insurance which will pay $200,000 at the end of the month of her death. Suppose that the de Moivre model holds with terminal age 120*. Find the mean and the standard deviation of the present value of this life insurance under the annual effective rate of interest of* $i = 10\%$.

Example 1

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Solution: The actuarial present value of this life insurance is

$$(200000)A_{40}^{(12)} = (200000)\sum_{j=1}^{\infty} \nu^{j/12} \cdot {}_{\frac{j-1}{12}|\frac{1}{12}}q_x$$

$$=(200000)\sum_{j=1}^{(12)(80)} (1.1)^{-j/12}\frac{1}{(12)(80)}$$

$$= = (200000)a_{\overline{960}|(1.1)^{1/12}-1}\frac{1}{960} = 26113.36354.$$

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Solution: The second moment of the present value of this life insurance is

$$(200000)^2 \cdot {}^2A_{40}^{(12)} = (200000)^2 \sum_{j=1}^{\infty} \nu^{2j/12} \cdot {}_{\frac{j-1}{12}|\frac{1}{12}}q_x$$

$$=(200000)^2 \sum_{j=1}^{(12)(80)} (1.1)^{-j/6} \frac{1}{(12)(80)}$$

$$= = (200000)^2 a_{\overline{960}|(1.1)^{1/6}-1} \frac{1}{960} = 2602235874.$$

Example 1

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Solution: The variance of the present value of this life insurance is

$$\mathrm{Var}(Z_{40}^{(12)}) = 2602235874 - (26113.36354)^2 = 1920328119.$$

The standard deviation of the present value of this life insurance is

$$\sqrt{1920328119} = 43821.54856.$$

