Manual for SOA Exam MLC.

Chapter 4. Life Insurance.

Section 4.7. Properties of the APV for continuous insurance.

Extract from:
"Arcones' Manual for the SOA Exam MLC. Fall 2009 Edition".
available at http://www.actexmadriver.com/

Properties of the APV for continuous insurance

The following table shows the definition of all the variables in the previous section:

type of insurance	payment
whole life insurance	$\overline{Z}_x = \nu^{T_x}$
n–year term life insurance	$\overline{Z}^1_{x:\overline{n}\vert} = \nu^{T_x} I(T_x \le n)$
n–year deferred life insurance	${}_{n\vert}\overline{Z}_x = \nu^{T_x} I(n < T_x)$
n–year pure endowment life insurance	$\overline{Z}_{x:\overline{n}\vert}^{\;\;1} = \nu^n I(n < T_x)$
n–year endowment life insurance	$\overline{Z}_{x:\overline{n}\vert} = \nu^{\min(T_x,n)}$

Theorem 1

We have that

$$\overline{Z}_x = \overline{Z}^1_{x:\overline{n}|} + {}_n|\overline{Z}_x,$$

$$\overline{A}_x = \overline{A}^1_{x:\overline{n}|} + {}_n|\overline{A}_x,$$

$${}^2\overline{A}_x = {}^2\overline{A}^1_{x:\overline{n}|} + {}^2{}_n|\overline{A}_x,$$

$$\mathrm{Var}(Z_x) = \mathrm{Var}(Z^1_{x:\overline{n}|}) + \mathrm{Var}({}_n|\overline{Z}_x) - 2\overline{A}^1_{x:\overline{n}|} \cdot {}_n|\overline{A}_x$$

$$\mathrm{Cov}(\overline{Z}^1_{x:\overline{n}|}, {}_n|\overline{Z}_x) = -\overline{A}^1_{x:\overline{n}|} \cdot {}_n|\overline{A}_x = \frac{1}{2}\left(\overline{A}^1_{x:\overline{n}|}{}^2 + {}_n|\overline{A}_x{}^2 - \overline{A}_x{}^2\right).$$

Theorem 2

We have that

$$\overline{Z}_{x:\overline{n}|} = \overline{Z}^{1}_{x:\overline{n}|} + \overline{Z}_{x:\overset{1}{\overline{n}|}},$$
$$\overline{A}_{x:\overline{n}|} = \overline{A}^{1}_{x:\overline{n}|} + \overline{A}_{x:\overset{1}{\overline{n}|}},$$
$${}^{2}\overline{A}_{x:\overline{n}|} = {}^{2}\overline{A}^{1}_{x:\overline{n}|} + {}^{2}\overline{A}_{x:\overset{1}{\overline{n}|}},$$
$$\mathrm{Var}(\overline{Z}_{x:\overline{n}|}) = \mathrm{Var}(\overline{Z}^{1}_{x:\overline{n}|}) + \mathrm{Var}(\overline{Z}_{x:\overset{1}{\overline{n}|}}) - 2\overline{A}^{1}_{x:\overline{n}|} \cdot \overline{A}_{x:\overset{1}{\overline{n}|}}$$
$$\mathrm{Cov}(\overline{Z}^{1}_{x:\overline{n}|}, \overline{Z}_{x:\overset{1}{\overline{n}|}}) = -\overline{A}^{1}_{x:\overline{n}|} \cdot \overline{A}_{x:\overset{1}{\overline{n}|}} = \frac{1}{2}\left(\overline{A}^{1}_{x:\overline{n}|}{}^{2} + \overline{A}_{x:\overset{1}{\overline{n}|}}{}^{2} - \overline{A}_{x:\overline{n}|}{}^{2}\right).$$

Theorem 3

We have that

$$ {}_{n|}\overline{A}_x = {}_nE_x \overline{A}_{x+n}. $$

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Proof: We have that

$$ {}_{n|}\overline{A}_x = \int_n^\infty \nu^t {}_tp_x \mu_{x+t}\, dt = \int_0^\infty \nu^{t+n} {}_{n+t}p_x \mu_{x+n+t}\, dt $$

$$ = \nu^n {}_np_x \int_0^\infty \nu^t {}_tp_{x+n} \mu_{x+n+t}\, dt = {}_nE_x \overline{A}_{x+n}. $$

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$$ = \nu^n {}_np_x \int_0^\infty \nu^t {}_tp_{x+n} \mu_{x+n+t}\, dt = {}_nE_x \overline{A}_{x+n}. $$

Corollary 1

$$ \overline{A}_x = \overline{A}^{1}_{x:\overline{n}|} + {}_nE_x \overline{A}_{x+n}. $$

Example 1

Suppose that $\delta = 0.04$ *and* (x) *has force of mortality* $\mu = 0.03$*. Calculate:*

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Calculate:
(i) $\overline{A}_x$ and $\mathrm{Var}(\overline{Z}_x)$.

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Suppose that $\delta = 0.04$ *and* (x) *has force of mortality* $\mu = 0.03$. *Calculate:*
(i) $\overline{A}_x$ and $\mathrm{Var}(\overline{Z}_x)$.

Solution: (i) We have that

$$\overline{A}_x = \frac{\mu}{\mu + \delta} = \frac{0.03}{0.03 + 0.04} = 0.4285714286,$$

$$^2\overline{A}_x = \frac{\mu}{\mu + 2\delta} = \frac{0.03}{0.03 + (2)(0.04)} = 0.2727272727,$$

$$\mathrm{Var}(\overline{Z}_x) = {}^2\overline{A}_x - (\overline{A}_x)^2 = 0.2727272727 - (0.428571428)^2$$
$$=0.0890538038.$$

Example 1

Suppose that $\delta = 0.04$ *and* (x) *has force of mortality* $\mu = 0.03$. *Calculate:*

(ii) $\overline{A}^{\,1}_{x:\overline{10|}}$ and $\mathrm{Var}(\overline{Z}^{\,1}_{x:\overline{10|}})$.

Example 1

Suppose that $\delta = 0.04$ *and* (x) *has force of mortality* $\mu = 0.03$.
Calculate:
(ii) $\overline{A}_{x:\overline{10|}}^{\,1}$ and $\mathrm{Var}(\overline{Z}_{x:\overline{10|}}^{\,1})$.

Solution: (ii) We have that

$$\overline{A}_{x:\overline{10|}}^{\,1} = e^{-10\delta}\,{}_{10}p_x = e^{-(10)(0.04)}e^{-(10)(0.03)} = e^{-0.7} = 0.4965853038,$$

$$^{2}\overline{A}_{x:\overline{10|}}^{\,1} = e^{-10(2)\delta}\,{}_{10}p_x = e^{-(10)(2)(0.04)}e^{-(10)(0.03)} = e^{-0.11}$$

$$=0.3328710837,$$

$$\mathrm{Var}(\overline{Z}_{x:\overline{10|}}^{\,1}) = {}^{2}\overline{A}_{x:\overline{10|}}^{\,1} - \overline{A}_{x:\overline{10|}}^{\,1\;2} = 0.3328710837 - (0.4965853038)^2$$

$$=0.08627411975.$$

Example 1

Suppose that $\delta = 0.04$ *and* (x) *has force of mortality* $\mu = 0.03$*.*
Calculate:
(iii) ${}_{10|}\overline{A}_x$ and $\mathrm{Var}({}_{10|}\overline{Z}_x)$.

Example 1

Suppose that $\delta = 0.04$ *and* (x) *has force of mortality* $\mu = 0.03$.
Calculate:
(iii) ${}_{10|}\overline{A}_x$ and $\mathrm{Var}({}_{10|}\overline{Z}_x)$.
Solution: (iii) We have that

$${}_{10|}\overline{A}_x = {}_nE_x \overline{A}_{x+n} = (0.4965853038)(0.4285714286) = 0.2128222731,$$

$${}^2{}_{10|}\overline{A}_x = {}^2{}_nE_x \cdot {}^2\overline{A}_{x+n} = (0.3328710837)(0.2727272727)$$

$$=0.09078302282,$$

$$\mathrm{Var}({}_{10|}\overline{Z}_x) = 0.09078302282 - (0.2128222731)^2 = 0.04548970289.$$

Example 1

Suppose that $\delta = 0.04$ *and* (x) *has force of mortality* $\mu = 0.03$*.*
Calculate:
(iv) $\overline{A}^{1}_{x:\overline{10}|}$ and $\mathrm{Var}(\overline{Z}^{1}_{x:\overline{10}|})$.

Example 1

Suppose that $\delta = 0.04$ *and* (x) *has force of mortality* $\mu = 0.03$. *Calculate:*

(iv) $\overline{A}^{1}_{x:\overline{10|}}$ and $\mathrm{Var}(\overline{Z}^{1}_{x:\overline{10|}})$.

Solution: (iv) We have that

$$\overline{A}^{1}_{x:\overline{10|}} = \overline{A}_x - {}_{10|}\overline{A}_x = 0.4285714286 - 0.2128222731 = 0.2157491555,$$

$${}^2\overline{A}^{1}_{x:\overline{10|}} = {}^2\overline{A}_x - {}^2{}_{10|}\overline{A}_x = 0.2727272727 - 0.09078302282$$

$$=0.1819442499,$$

$$\mathrm{Var}(\overline{Z}^{1}_{x:\overline{10|}}) = {}^2\overline{A}^{1}_{x:\overline{10|}} - \overline{A}^{1}_{x:\overline{10|}}{}^{2} = 0.1819442499 - (0.2157491555)^2$$

$$=0.1353965518.$$

Example 1

Suppose that $\delta = 0.04$ *and* (x) *has force of mortality* $\mu = 0.03$*. Calculate:*
(v) $\overline{A}_{x:\overline{10}|}$ and $\mathrm{Var}(\overline{Z}_{x:\overline{10}|})$.

Example 1

Suppose that $\delta = 0.04$ *and* (x) *has force of mortality* $\mu = 0.03$.
Calculate:
(v) $\overline{A}_{x:\overline{10}|}$ and $\mathrm{Var}(\overline{Z}_{x:\overline{10}|})$.
Solution: (v) We have that

$$\overline{A}_{x:\overline{10}|} = \overline{A}^{1}_{x:\overline{10}|} + \overline{A}_{x:\overset{1}{\overline{10}|}} = 0.2157491555 + 0.4965853038$$
$$=0.7123344593,$$
$$^{2}\overline{A}_{x:\overline{10}|} = {}^{2}\overline{A}^{1}_{x:\overline{10}|} + {}^{2}\overline{A}_{x:\overset{1}{\overline{10}|}} = 0.1819442499 + 0.3328710837$$
$$=0.5148153336,$$
$$\mathrm{Var}(\overline{Z}^{1}_{x:\overline{10}|}) = {}^{2}\overline{A}^{1}_{x:\overline{10}|} - \overline{A}^{1}_{x:\overline{10}|}{}^{2} = 0.5148153336 - (0.7123344593)^2$$
$$=0.007394951694.$$

Theorem 4

Suppose that the survival function is $s(x) = e^{-\mu x}$, $0 \leq x$. *Then,*

(i) $\overline{A}_x = \frac{\mu}{\mu+\delta}$.

(ii) $\overline{A}_{x:\overset{1}{\overline{n}|}} = e^{-n(\mu+\delta)}$.

(iii) ${}_{n|}\overline{A}_x = e^{-n(\mu+\delta}\frac{\mu}{\mu+\delta}$.

(iv) $\overline{A}^{1}_{x:\overline{n}|} = (1 - e^{-n(\mu+\delta)})\frac{\mu}{\mu+\delta}$.

(v) $\overline{A}_{x:\overline{n}|} = (1 - e^{-n(\mu+\delta)})\frac{\mu}{\mu+\delta} + e^{-n(\mu+\delta)}$.

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Proof: (i) follows from a previous theorem.

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(v) $\overline{A}_{x:\overline{n}|} = (1 - e^{-n(\mu+\delta)})\frac{\mu}{\mu+\delta} + e^{-n(\mu+\delta)}$.

Proof: (ii)

$$\overline{A}_{x:\overline{n}|}^{\;\;1} = e^{-n\delta} \cdot {}_{n}p_x = e^{-n(\mu+\delta)}.$$

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(v) $\overline{A}_{x:\overline{n}|} = (1 - e^{-n(\mu+\delta)})\frac{\mu}{\mu+\delta} + e^{-n(\mu+\delta)}$.

Proof: (iii)

$$ {}_{n|}\overline{A}_x = {}_nE_x\overline{A}_{x+n} = e^{-n(\mu+\delta)}\frac{\mu}{\mu+\delta}. $$

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(iv) $\overline{A}^{1}_{x:\overline{n}|} = (1 - e^{-n(\mu+\delta)})\frac{\mu}{\mu+\delta}$.
(v) $\overline{A}_{x:\overline{n}|} = (1 - e^{-n(\mu+\delta)})\frac{\mu}{\mu+\delta} + e^{-n(\mu+\delta)}$.

Proof: (iv)

$$\overline{A}^{1}_{x:\overline{n}|} = \overline{A}_x - {}_{n|}\overline{A}_x = \frac{\mu}{\mu+\delta} - e^{-n(\mu+\delta)}\frac{\mu}{\mu+\delta}$$
$$=(1 - e^{-n(\mu+\delta)})\frac{\mu}{\mu+\delta}.$$

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(iv) $\overline{A}^{1}_{x:\overline{n}|} = (1 - e^{-n(\mu+\delta)})\frac{\mu}{\mu+\delta}$.
(v) $\overline{A}_{x:\overline{n}|} = (1 - e^{-n(\mu+\delta)})\frac{\mu}{\mu+\delta} + e^{-n(\mu+\delta)}$.

Proof: (v)

$$\overline{A}_{x:\overline{n}|} = \overline{A}^{1}_{x:\overline{n}|} + \overline{A}_{x:\overset{1}{\overline{n}|}} = (1 - e^{-n(\mu+\delta)})\frac{\mu}{\mu+\delta} + e^{-n(\mu+\delta)}.$$

Example 2

Find the APV of a 15*-year term life insurance to* (x) *with unity payment if* $\delta = 0.06$ *and* (x) *has a constant force of mortality* $\mu = 0.05$*.*

Example 2

Find the APV of a 15-year term life insurance to (x) with unity payment if $\delta = 0.06$ and (x) has a constant force of mortality $\mu = 0.05$.

Solution: We have that

$$\overline{A}^{1}_{x:\overline{15}|} = \frac{(0.05)(1 - e^{-(15)(0.05+0.06)})}{0.05 + 0.06} = \frac{(0.05)(1 - e^{-(15)(0.11)})}{0.11}$$

$$=0.3672500415.$$

Theorem 5

Assume de Moivre's law with terminal age ω and that ω and x are positive integers. Then,

(i) $\overline{A}_x = \frac{\overline{a}_{\overline{\omega-x}|i}}{\omega-x}$.

(ii) $\overline{A}_{x:\overline{n}|}^{\ \ 1} = e^{-n\delta}\frac{\omega-x-n}{\omega-x}$.

(iii) ${}_{n|}\overline{A}_x = e^{-n\delta}\frac{\overline{a}_{\overline{\omega-x-n}|i}}{\omega-x}$.

(iv) $\overline{A}^{1}_{x:\overline{n}|} = \frac{\overline{a}_{\overline{n}|i}}{\omega-x}$.

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(iii) ${}_{n|}\overline{A}_x = e^{-n\delta}\frac{\overline{a}_{\overline{\omega-x-n}|i}}{\omega-x}$.

(iv) $\overline{A}^{1}_{x:\overline{n}|} = \frac{\overline{a}_{\overline{n}|i}}{\omega-x}$.

(v) $\overline{A}_{x:\overline{n}|} = \frac{\overline{a}_{\overline{n}|i}}{\omega-x} + e^{-n\delta}\frac{\omega-x-n}{\omega-x}$.

Proof: (i) It follows from a previous theorem.

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(iii) ${}_n|\overline{A}_x = e^{-n\delta}\frac{\overline{a}_{\overline{\omega-x-n}|i}}{\omega-x}$.

(iv) $\overline{A}^{1}_{x:\overline{n}|} = \frac{\overline{a}_{\overline{n}|i}}{\omega-x}$.

(v) $\overline{A}_{x:\overline{n}|} = \frac{\overline{a}_{\overline{n}|i}}{\omega-x} + e^{-n\delta}\frac{\omega-x-n}{\omega-x}$.

Proof: (ii)

$$\overline{A}_{x:\overline{n}|}^{\,1} = e^{-n\delta}\cdot {}_np_x = e^{-n\delta}\frac{\omega-x-n}{\omega-x}.$$

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(iv) $\overline{A}^{1}_{x:\overline{n}|} = \frac{\overline{a}_{\overline{n}|i}}{\omega-x}$.

(v) $\overline{A}_{x:\overline{n}|} = \frac{\overline{a}_{\overline{n}|i}}{\omega-x} + e^{-n\delta}\frac{\omega-x-n}{\omega-x}$.

Proof: (iii)

$${}_{n|}\overline{A}_x = {}_nE_x\overline{A}_{x+n} = e^{-n\delta}\frac{\omega-x-n}{\omega-x}\frac{\overline{a}_{\overline{\omega-x-n}|i}}{\omega-x-n} = e^{-n\delta}\frac{\overline{a}_{\overline{\omega-x-n}|i}}{\omega-x}.$$

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(iv) $\overline{A}^{1}_{x:\overline{n}|} = \frac{\overline{a}_{\overline{n}|i}}{\omega-x}$.

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(v) $\overline{A}_{x:\overline{n}|} = \frac{\overline{a}_{\overline{n}|i}}{\omega-x} + e^{-n\delta}\frac{\omega-x-n}{\omega-x}$.

Proof: (v)

$$\overline{A}_{x:\overline{n}|} = \overline{A}^{1}_{x:\overline{n}|} + \overline{A}_{x:\overline{n}|}^{\;\;1} = \frac{\overline{a}_{\overline{n}|i}}{\omega - x} + e^{-n\delta}\frac{\omega - x - n}{\omega - x}.$$

Example 3

Julia is 40 year old. She buys a 15-year term deferred life policy insurance which will pay \$50,000 at the time of her death. Suppose that the survival function is $s(x) = 1 - \frac{x}{100}$, $0 \leq x \leq 100$. *Suppose that the continuously compounded rate of interest is* $\delta = 0.05$.

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(i) Find the actuarial present value of the benefit of this life insurance.

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(i) Find the actuarial present value of the benefit of this life insurance.

Solution: (i) The density of T_{40} is $f_{T_{40}}(t) = \frac{1}{60}$, $0 \le t \le 60$. Hence,

$$
{}_{15|}\overline{A}_x = e^{-(15)(0.05)} \frac{\overline{a}_{\overline{60-15}|i}}{60} = e^{-(15)(0.05)} \frac{1 - e^{-(45)(0.05)}}{(0.05)(60)}
$$

$$
= \frac{e^{-0.75} - e^{-3}}{3} = 0.1408598281.
$$

The actuarial present value is $(50000)(0.1408598281) = 7042.991405$.

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(ii) Find the standard deviation of the present value of the benefit of this life insurance.

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(ii) Find the standard deviation of the present value of the benefit of this life insurance.

Solution: (ii) We have that

$$ {}^{2}{}_{15|}\overline{A}_x = e^{-(15)(2)(0.05)} \frac{\overline{a}_{\overline{60-15}|(1+i)^2-1}}{60} = e^{-(15)(0.05)} \frac{1 - e^{-(45)(2)(0.05)}}{(2)(0.05)(60)} $$

$$ = \frac{e^{-1.5} - e^{-6}}{6} = 0.03677523466, $$

$$ \mathrm{Var}(50000 \cdot {}_{15|}\overline{Z}_x) = (50000)^2(0.03677523466 - (0.1408598281)^2) $$

$$ = 42334358.72. $$

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(ii) Find the standard deviation of the present value of the benefit of this life insurance.

Solution: (ii)The standard deviation of $50000 \cdot {}_{15|}\overline{Z}_x$ is $\sqrt{42334358.72} = 6506.485896$.

Example 4

Julia is 40 year old. She buys a 15–year endowment policy insurance which will pay \$50,000 at the time of her death, or in 15 years whatever comes first. Suppose that the survival function is $s(x) = 1 - \frac{x}{100}$, $0 \leq x \leq 100$. *Suppose that the continuously compounded rate of interest is* $\delta = 0.05$.

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(i) Find the actuarial present value of the benefit of this life insurance.

Solution: (i) The density of T_{40} is $f_{T_{40}}(t) = \frac{1}{60}$, $0 \le t \le 60$. Hence,

$$\overline{A}_{40:\overline{15}|} = \frac{\overline{a}_{\overline{15}|i}}{100-40} + e^{-(15)(0.05)}\frac{100-40-15}{100-40}$$

$$= \frac{1-e^{-(15)(0.05)}}{(0.05)(60)} + e^{-0.75}\frac{45}{60} = \frac{1-e^{-0.75}}{3} + e^{-0.75}(0.75) = 0.530152730$$

The actuarial present value is $(50000)(0.5301527303) = 26507.63652$.

Example 4

Julia is 40 year old. She buys a 15–year endowment policy insurance which will pay \$50,000 at the time of her death, or in 15 years whatever comes first. Suppose that the survival function is $s(x) = 1 - \frac{x}{100}$, $0 \le x \le 100$. *Suppose that the continuously compounded rate of interest is* $\delta = 0.05$.

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Solution: We have that

$$
\begin{aligned}
{}^2\overline{A}_{40:\overline{15}|} &= \frac{\overline{a}_{\overline{15}|(1+i)^2-1}}{100-40} + e^{-(15)(2)(0.05)}\frac{100-40-15}{100-40} \\
&= \frac{1-e^{-(15)(2)(0.05)}}{(2)(0.05)(60)} + e^{-1.5}\frac{45}{60} = \frac{1-e^{-1.5}}{6} + e^{-1.5}(0.75) \\
&= 0.2968259268, \\
\mathrm{Var}(50000\overline{Z}_{40:\overline{15}|}) &= (50000)^2(0.2968259268 - (0.5301527303)^2) \\
&= 39410023.39.
\end{aligned}
$$

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Solution: The standard deviation of $50000\overline{Z}_{40:\overline{15|}}$ is $\sqrt{39410023.39} = 6277.740309$.

