Manual for SOA Exam MLC.

Chapter 4. Life Insurance.

Section 4.8. Non–level payments paid at the time of death.

Extract from:
"Arcones' Manual for the SOA Exam MLC. Fall 2009 Edition".
available at http://www.actexmadriver.com/

Non–level payments paid at the time of death

Suppose that if failure happens at time t, then the benefit payment is b_t. The present value of the benefit payment is denoted by $\overline{B}_x = b_{T_x}\nu^{T_x}$. The actuarial present value of this benefit is

$$E[\overline{B}_x] = \int_0^\infty b_t \nu^t f_{T_x}(t)\,dt = \int_0^\infty b_t \nu^t \cdot {}_tp_x \mu_{x+t}\,dt.$$

Example 1

For a whole life insurance on (60), *you are given:*
(i) Death benefits are paid at the moment of death.
(ii) Mortality follows a de Moivre model with terminal age 100.
(iii) $i = 7\%$.
(iv) $b_t = (20000)(1.04)^t$, $t \geq 0$.
Calculate the mean and the standard deviation of the present value random variable for this insurance.

Solution: The present value random variable is

$$Z = b_{T_{60}} \nu^{T_{60}} = (20000)(1.04)^{T_{60}}(1.07)^{-T_{60}} = (20000)\left(\frac{1.04}{1.07}\right)^{T_{60}}.$$

The density of T_{60} is ${}_tp_{60}\mu_{60}(t) = \frac{1}{40}$, $0 \leq t \leq 40$.

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The density of T_{60} is

$${}_tp_{60}\mu_{60}(t) = \frac{1}{40},\ 0 \le t \le 40.$$

Hence,

$$E[Z] = \int_0^{40} (20000)\left(\frac{1.04}{1.07}\right)^t \frac{1}{40}\,dt = \frac{(20000)((\frac{1.04}{1.07})^{40} - 1)}{40\ln(1.04/1.07)}$$

$$= 11945.06573,$$

$$E[Z^2] = \int_0^{40} (20000)^2\left(\frac{1.04}{1.07}\right)^{2t} \frac{1}{40}\,dt = \frac{(20000)^2((\frac{1.04}{1.07})^{80} - 1)}{80\ln(1.04/1.07)}$$

$$= 157748208.7,$$

$$\mathrm{Var}(Z) = 157748208.7 - (11945.06573)^2 = 15063613.41,$$

$$\sqrt{\mathrm{Var}(Z)} = \sqrt{15063613.41} = 3881.187114.$$

If $b_t = t$, $t \geq 0$, we say that we have an increasing life insurance in the continuous case. Its actuarial present value is denoted by

$$(\bar{I}\,\bar{A})_x = \int_0^\infty t\nu^t \cdot {}_t p_x \mu_{x+t}\, dt.$$

Theorem 1

Under constant force of mortality μ,

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Proof:

$$(\bar{I}\,\bar{A})_x = \int_0^\infty tv^t \cdot {}_tp_x\mu_{x+t}\,dt = \int_0^\infty te^{-\delta t}\mu e^{-\mu t}\,dt$$
$$= \mu\int_0^\infty te^{-(\delta+\mu)t}\,dt = \frac{\mu}{(\mu+\delta)^2}.$$

Let $\lceil t \rceil$ be the **least integer greater than or equal to** t. $\lceil t \rceil$ is called the **ceiling** of t. We have that $\lceil t \rceil = k$ if k is an integer such that $k - 1 < t \leq k$. Notice that

$$\lceil t \rceil = \begin{cases} 1 & \text{if } 0 < t \leq 1, \\ 2 & \text{if } 1 < t \leq 2, \\ 3 & \text{if } 2 < t \leq 3, \\ . & \dots \\ . & \dots \end{cases}$$

If $b_t = \lceil t \rceil$, $t \geq 0$, we say that we have an increasing life insurance in the piecewise–continuous case. Its actuarial present value is denoted by

$$(I\,\overline{A})_x = \int_0^\infty \lceil t \rceil \nu^t \cdot {}_tp_x \mu_{x+t}\, dt = \sum_{k=1}^{\infty} \int_{(k-1,k]} k\nu^t \cdot {}_tp_x \mu_{x+t}\, dt.$$

If $b_t = t$, $0 \le t \le n$, zero otherwise, we say that we have an n–year term increasing life insurance in the continuous case. Its actuarial present value is denoted by

$$(\bar{I}\ \bar{A})^{1}_{x:\overline{n}|} = \int_0^n t\nu^t \cdot {}_tp_x \mu_{x+t}\, dt.$$

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If $b_t = n - t$, $0 \le t \le n$, zero otherwise, we say that we have an n–year term decreasing life insurance in the continuous case. Its actuarial present value is denoted by

$$\left(\overline{D}\ \overline{A}\right)^{1}_{x:\overline{n}|} = \int_0^n (n-t)\nu^t \cdot {}_t p_x \mu_{x+t}\, dt.$$

If $b_t = \lceil n - t \rceil$, $0 \le t \le n$, zero otherwise, we say that we have an n–year term decreasing life insurance in the piecewise–continuous case. Its actuarial present value is denoted by

$$\left(D\,\overline{A}\right)^{1}_{x:\overline{n}|} = \int_0^n \lceil n - t \rceil \nu^t \cdot {}_t p_x \mu_{x+t}\, dt.$$

