Manual for SOA Exam MLC.

Chapter 4. Life Insurance.

Section 4.9. Computing APV's from a life table.

Extract from:
"Arcones' Manual for the SOA Exam MLC. Fall 2009 Edition".
available at http://www.actexmadriver.com/

Computing APV's from a life table

Usually, survival functions do not have an analytical form. In this section, we will consider the computation of some of the present values of life insurance products using a life table. Suppose that ℓ_x is known whenever x is a nonnegative integer. Then, we can find ${}_np_x$, whenever x and n are integers.

Assuming a uniform distribution of deaths over each year of death, $K(x)$ and $S_x = T_x - K(x)$ are independent r.v.'s and S_x has a distribution uniform on $(0, 1)$. Hence, K_x and S_x are independent r.v.'s and $T_x = K(x) + S_x = K_x + S_x - 1$.

Theorem 1

Assume a uniform distribution of deaths over each year of death. Suppose that b_t, $t \geq 0$, is constant in each interval $(k-1, k]$, $k = 1, 2, \dots$. Then,

$$E[b_{T_x} \nu^{T_x}] = E[b_{K_x} \nu^{K_x}] \frac{i}{\delta}.$$

Theorem 1

Assume a uniform distribution of deaths over each year of death. Suppose that b_t, $t \geq 0$, is constant in each interval $(k-1, k]$, $k = 1, 2, \dots$. Then,

$$E[b_{T_x}\nu^{T_x}] = E[b_{K_x}\nu^{K_x}]\frac{i}{\delta}.$$

Proof: Since S_x has a distribution uniform on $(0, 1)$, so has $1 - S_x$. So,

$$E[\nu^{S_x-1}] = E[e^{\delta(1-S_x)}] = \int_0^1 e^{\delta t}\,dt = \frac{e^{\delta}-1}{\delta} = \frac{i}{\delta}.$$

Hence,

$$E[b_{T_x}\nu^{T_x}] = E[b_{K_x}\nu^{T_x}] = E[b_{K_x}\nu^{K_x+S_x-1}]$$
$$= E[b_{K_x}\nu^{K_x}]E[\nu^{S_x-1}] = E[b_{K_x}\nu^{K_x}]\frac{i}{\delta}$$

Theorem 1

Assume a uniform distribution of deaths over each year of death. Suppose that b_t, $t \geq 0$, is constant in each interval $(k-1, k]$, $k = 1, 2, \ldots$. Then,

$$E[b_{T_x} \nu^{T_x}] = E[b_{K_x} \nu^{K_x}] \frac{i}{\delta}.$$

The factor $\frac{i}{\delta}$ has a life table interpretation. Suppose that a death happens during the k–th year. Under the discrete life insurance a benefit payment of one is made at time k. Under the continuous life insurance and an uniform distribution of deaths, this death can happen uniformly on the interval $[k-1, k]$. So, for each dollar paid at time k in the discrete case, we get a continuous cashflow of a unit rate over the interval $[k-1, k]$ in the continuous case. The present value at time k of this continuous cashflow is

$$\overline{s}_{\overline{1}|i} = \frac{(1+i)^1 - 1}{\delta} = \frac{i}{\delta}.$$

Theorem 2

Assuming a uniform distribution of deaths, we have that:
(i) $\overline{A}_x = \frac{i}{\delta} A_x$.
(ii) $\overline{A}^{1}_{x:\overline{n}|} = \frac{i}{\delta} A^{1}_{x:\overline{n}|}$.
(iii) ${}_{n}|\overline{A}_x = \frac{i}{\delta} \cdot {}_{n}|A_x$.
(iv) $\overline{A}_{x:\overline{n}|} = \frac{i}{\delta} A^{1}_{x:\overline{n}|} + A_{x:\overset{1}{\overline{n}|}}$.

Proof: (i) We apply Theorem 1 with $b_t = 1$, $t \geq 0$. We have that $\overline{A}_x = E[\nu^{T_x}] = E[b_{T_x}\nu^{T_x}]$ and $A_x = E[\nu^{K_x}] = E[b_{K_x}\nu^{K_x}]$.
(ii) We apply Theorem 1 with $b_t = I(t \leq n)$, $t \geq 0$. We have that
$\overline{A}^{1}_{x:\overline{n}|} = E[I(T_x \leq n)\nu^{T_x}] = E[b_{T_x}\nu^{T_x}]$ and
$A^{1}_{x:\overline{n}|} = E[I(K_x \leq n)\nu^{K_x}] = E[b_{K_x}\nu^{K_x}]$.
(iii) We apply Theorem 1 with $b_t = I(t > n)$, $t \geq 0$.
(iv)

$$\overline{A}_{x:\overline{n}|} = \overline{A}^{1}_{x:\overline{n}|} + \overline{A}_{x:\overset{1}{\overline{n}|}} = \frac{i}{\delta} A^{1}_{x:\overline{n}|} + A_{x:\overset{1}{\overline{n}|}}.$$

Example 1

Assuming a uniform distribution of deaths, $i = 6\%$ *and the life table in the manual, find:* $\overline{A}_{50}$, ${}_{15}E_{50}$, ${}_{15|}\overline{A}_{50}$, $\overline{A}^{1}_{50:\overline{15}|}$ *and* $\overline{A}_{50:\overline{15}|}$.

Example 1

Assuming a uniform distribution of deaths, $i = 6\%$ and the life table in the manual, find: $\overline{A}_{50}$, ${}_{15}E_{50}$, ${}_{15|}\overline{A}_{50}$, $\overline{A}^{\;1}_{50:\overline{15}|}$ and $\overline{A}_{50:\overline{15}|}$.

Solution: We have that

$$\overline{A}_{50} = \frac{i}{\delta}A_{50} = \frac{0.06}{\ln(1.06)}0.20696 = 0.2131085067,$$

$${}_{15}E_{50} = \nu^{15}\,{}_{15}p_{50} = (1.06)^{-15}\frac{\ell_{65}}{\ell_{50}} = (1.06)^{-15}\frac{83114}{93735} = 0.3699852591,$$

$$\overline{A}_{65} = \frac{i}{\delta}A_{65} = \frac{0.06}{\ln(1.06)}0.3761 = 0.3872734315,$$

$${}_{15|}\overline{A}_{50} = {}_{15}E_{50}\overline{A}_{65} = (0.3699852591)(0.3872734315) = 0.1432854609,$$

$$\overline{A}^{\;1}_{50:\overline{15}|} = \overline{A}_{65} - {}_{15|}\overline{A}_{50} = 0.3872734315 - 0.1432854609$$

$$=0.2439879706,$$

$$\overline{A}_{50:\overline{15}|} = \overline{A}^{\;1}_{50:\overline{15}|} + {}_{15}E_{50} = 0.2439879706 + 0.3699852591$$

$$=0.6139732297.$$

Example 2

Suppose that $i = 0.05$, $q_x = 0.03$ *and* $q_{x+1} = 0.04$. *Assuming a uniform distribution of deaths, find* $\overline{A}^{1}_{x:\overline{2}|}$ *and* $\mathrm{Var}(\overline{Z}^{1}_{x:\overline{2}|})$.

Example 2

Suppose that $i = 0.05$, $q_x = 0.03$ and $q_{x+1} = 0.04$. Assuming a uniform distribution of deaths, find $\overline{A}^{1}_{x:\overline{2}|}$ and $\mathrm{Var}(\overline{Z}^{1}_{x:\overline{2}|})$.

Solution: We have that

$$A^{1}_{x:\overline{2}|} = \nu q_x + \nu^2 p_x q_{x+1} = (1.05)^{-1}(0.03) + (1.05)^{-2}(0.97)(0.04)$$
$$= 0.06376417234,$$

$$\overline{A}^{1}_{x:\overline{2}|} = \frac{i}{\delta} A^{1}_{x:\overline{2}|} = \frac{0.05}{\ln(1.05)} 0.06376417234 = 0.03825829018,$$

$$^{2}A^{1}_{x:\overline{2}|} = \nu^2 q_x + \nu^4 p_x q_{x+1} = (1.05)^{-2}(0.03) + (1.05)^{-4}(0.97)(0.04)$$
$$= 0.05913174038,$$

$$^{2}\overline{A}^{1}_{x:\overline{2}|} = \frac{((1+i)^2 - 1)^2 A^{1}_{x:\overline{2}|}}{2\delta} = \frac{((1.05)^2 - 1)(0.0591317)}{2\ln(1.05)} = 0.0621129$$

$$\mathrm{Var}(\overline{Z}^{1}_{x:\overline{2}|}) = {}^{2}\overline{A}^{1}_{x:\overline{2}|} - (A^{1}_{x:\overline{2}|})^2 = 0.06211296367 - (0.03825829018)^2$$
$$= 0.0606492669.$$

Example 3

Consider the life table

x	80	81	82	83	84	85	86
ℓ_x	250	217	161	107	62	28	0

Suppose that $i = 6.5\%$. *Calculate:*
(i) A_{80}. *(ii)* $A_{80:\overset{1}{\overline{3}|}}$. *(iii)* ${}_{3|}A_{80}$. *(iv)* $A^{1}_{80:\overline{3}|}$. *(v)* $A_{80:\overline{3}|}$. *(vi)* $\overline{A}_{80}$. *(vii)* $\overline{A}_{80:\overset{1}{\overline{3}|}}$. *(viii)* ${}_{3|}\overline{A}_{80}$. *(ix)* $\overline{A}^{1}_{80:\overline{3}|}$. *(x)* $\overline{A}_{80:\overline{3}|}$.

Example 3

Consider the life table

x	80	81	82	83	84	85	86
ℓ_x	250	217	161	107	62	28	0

Suppose that $i = 6.5\%$. *Calculate:*
(i) A_{80}. *(ii)* $A_{80:\overline{3}|}^{\,\,1}$. *(iii)* ${}_{3|}A_{80}$. *(iv)* $A^1_{80:\overline{3}|}$. *(v)* $A_{80:\overline{3}|}$. *(vi)* $\overline{A}_{80}$. *(vii)* $\overline{A}_{80:\overline{3}|}^{\,\,1}$. *(viii)* ${}_{3|}\overline{A}_{80}$. *(ix)* $\overline{A}^1_{80:\overline{3}|}$. *(x)* $\overline{A}_{80:\overline{3}|}$.

Solution: (i) $A_{80} = \sum_{k=1}^{\infty} \nu^k \frac{\ell_{80+k-1} - \ell_{80+k}}{\ell_{80}}$ is

$$
\begin{aligned}
&(1.065)^{-1}\frac{250-217}{250} + (1.065)^{-2}\frac{217-161}{250} + (1.065)^{-3}\frac{161-107}{250} \\
&+(1.065)^{-4}\frac{107-62}{250} + (1.065)^{-5}\frac{62-28}{250} + (1.065)^{-6}\frac{28-0}{250} \\
&=0.12394366197 + 0.19749167934 + 0.17881540383 + 0.13991815636 \\
&\quad + 0.09926379377 + 0.07675742131 = 0.8161901166.
\end{aligned}
$$

Example 3

Consider the life table

x	80	81	82	83	84	85	86
ℓ_x	250	217	161	107	62	28	0

Suppose that $i = 6.5\%$. *Calculate:*
(i) A_{80}. *(ii)* $A_{80:\overline{3}|}^{\;\;\;1}$. *(iii)* ${}_3|A_{80}$. *(iv)* $A^1_{80:\overline{3}|}$. *(v)* $A_{80:\overline{3}|}$. *(vi)* $\overline{A}_{80}$. *(vii)* $\overline{A}_{80:\overline{3}|}^{\;\;\;1}$. *(viii)* ${}_3|\overline{A}_{80}$. *(ix)* $\overline{A}^1_{80:\overline{3}|}$. *(x)* $\overline{A}_{80:\overline{3}|}$.

Solution: (ii)

$$A_{80:\overline{3}|}^{\;\;\;1} = \nu^3 \frac{\ell_{80+3}}{\ell_{80}} = (1.065)^{-3} \frac{107}{250} = 0.35431941129.$$

Example 3

Consider the life table

x	80	81	82	83	84	85	86
ℓ_x	250	217	161	107	62	28	0

Suppose that $i = 6.5\%$. *Calculate:*
(i) A_{80}. *(ii)* $A_{80:\overline{3}|}^{\;\;1}$. *(iii)* ${}_{3|}A_{80}$. *(iv)* $A^{1}_{80:\overline{3}|}$. *(v)* $A_{80:\overline{3}|}$. *(vi)* $\overline{A}_{80}$. *(vii)* $\overline{A}_{80:\overline{3}|}^{\;\;1}$. *(viii)* ${}_{3|}\overline{A}_{80}$. *(ix)* $\overline{A}^{1}_{80:\overline{3}|}$. *(x)* $\overline{A}_{80:\overline{3}|}$.

Solution: (iii)

$$
\begin{aligned}
{}_{3|}A_{80} &= \sum_{k=4}^{\infty} \nu^k \frac{\ell_{80+k-1} - \ell_{80+k}}{\ell_{80}} \\
=&(1.065)^{-4}\frac{107-62}{250} + (50000)(1.065)^{-5}\frac{62-28}{250} + (1.065)^{-6}\frac{28-0}{250} \\
=&0.13991815636 + 0.09926379377 + 0.07675742131 = 0.3159393714.
\end{aligned}
$$

Example 3

Consider the life table

x	80	81	82	83	84	85	86
ℓ_x	250	217	161	107	62	28	0

Suppose that $i = 6.5\%$. *Calculate:*
(i) A_{80}. *(ii)* $A_{80:\overline{3}|}^{\ \ 1}$. *(iii)* ${}_{3|}A_{80}$. *(iv)* $A^1_{80:\overline{3}|}$. *(v)* $A_{80:\overline{3}|}$. *(vi)* $\overline{A}_{80}$. *(vii)* $\overline{A}_{80:\overline{3}|}^{\ \ 1}$. *(viii)* ${}_{3|}\overline{A}_{80}$. *(ix)* $\overline{A}^1_{80:\overline{3}|}$. *(x)* $\overline{A}_{80:\overline{3}|}$.

Solution: (iv)

$$A^1_{80:\overline{3}|} = A_{80} - {}_{3|}A_{80} = 0.8161901166 - 0.3159393714 = 0.5002507452$$

Example 3

Consider the life table

x	80	81	82	83	84	85	86
ℓ_x	250	217	161	107	62	28	0

Suppose that $i = 6.5\%$. *Calculate:*
(i) A_{80}. *(ii)* $A_{80:\overset{1}{\overline{3}|}}$. *(iii)* ${}_3|A_{80}$. *(iv)* $A^1_{80:\overline{3}|}$. *(v)* $A_{80:\overline{3}|}$. *(vi)* $\overline{A}_{80}$. *(vii)* $\overline{A}_{80:\overset{1}{\overline{3}|}}$. *(viii)* ${}_3|\overline{A}_{80}$. *(ix)* $\overline{A}^1_{80:\overline{3}|}$. *(x)* $\overline{A}_{80:\overline{3}|}$.

Solution: (v)

$$A_{80:\overline{3}|} = A^1_{80:\overline{3}|} + A_{80:\overset{1}{\overline{3}|}} = 0.5002507452 + 0.35431941129$$

$$= 0.8545701565.$$

Example 3

Consider the life table

x	80	81	82	83	84	85	86
ℓ_x	250	217	161	107	62	28	0

Suppose that $i = 6.5\%$. *Calculate:*
(i) A_{80}. *(ii)* $A_{80:\overset{1}{\overline{3}|}}$. *(iii)* ${}_{3|}A_{80}$. *(iv)* $A^{1}_{80:\overline{3}|}$. *(v)* $A_{80:\overline{3}|}$. *(vi)* $\overline{A}_{80}$. *(vii)* $\overline{A}_{80:\overset{1}{\overline{3}|}}$. *(viii)* ${}_{3|}\overline{A}_{80}$. *(ix)* $\overline{A}^{1}_{80:\overline{3}|}$. *(x)* $\overline{A}_{80:\overline{3}|}$.

Solution: (vi)

$$\overline{A}_{80} = \frac{i}{\delta} A_{80} = \frac{0.065}{\ln(1.065)}(0.8161901166) = 0.8424379003.$$

Example 3

Consider the life table

x	80	81	82	83	84	85	86
ℓ_x	250	217	161	107	62	28	0

Suppose that $i = 6.5\%$. *Calculate:*
(i) A_{80}. *(ii)* $A_{80:\overline{3}|}^{\;\;\;\;\;1}$. *(iii)* ${}_{3|}A_{80}$. *(iv)* $A^{1}_{80:\overline{3}|}$. *(v)* $A_{80:\overline{3}|}$. *(vi)* $\overline{A}_{80}$. *(vii)* $\overline{A}_{80:\overline{3}|}^{\;\;\;\;\;1}$. *(viii)* ${}_{3|}\overline{A}_{80}$. *(ix)* $\overline{A}^{1}_{80:\overline{3}|}$. *(x)* $\overline{A}_{80:\overline{3}|}$.

Solution: (vii)

$$\overline{A}_{80:\overline{3}|}^{\;\;\;\;\;1} = A_{80:\overline{3}|}^{\;\;\;\;\;1} = 0.35431941129.$$

Example 3

Consider the life table

x	80	81	82	83	84	85	86
ℓ_x	250	217	161	107	62	28	0

Suppose that $i = 6.5\%$. *Calculate:*
(i) A_{80}. *(ii)* $A_{80:\overset{1}{\overline{3}|}}$. *(iii)* ${}_{3|}A_{80}$. *(iv)* $A^{1}_{80:\overline{3}|}$. *(v)* $A_{80:\overline{3}|}$. *(vi)* $\overline{A}_{80}$. *(vii)* $\overline{A}_{80:\overset{1}{\overline{3}|}}$. *(viii)* ${}_{3|}\overline{A}_{80}$. *(ix)* $\overline{A}^{1}_{80:\overline{3}|}$. *(x)* $\overline{A}_{80:\overline{3}|}$.

Solution: (viii)

$$ {}_{3|}\overline{A}_{80} = \frac{i}{\delta} \cdot {}_{3|}A_{80} = \frac{0.065}{\ln(1.065)}(0.3159393714) = 0.3260996369. $$

Example 3

Consider the life table

x	80	81	82	83	84	85	86
ℓ_x	250	217	161	107	62	28	0

Suppose that $i = 6.5\%$. *Calculate:*
(i) A_{80}. *(ii)* $A_{80:\overline{3}|}^{\ \ \frac{1}{}}$. *(iii)* ${}_{3|}A_{80}$. *(iv)* $A^{1}_{80:\overline{3}|}$. *(v)* $A_{80:\overline{3}|}$. *(vi)* $\overline{A}_{80}$. *(vii)* $\overline{A}_{80:\overline{3}|}^{\ \ \frac{1}{}}$. *(viii)* ${}_{3|}\overline{A}_{80}$. *(ix)* $\overline{A}^{1}_{80:\overline{3}|}$. *(x)* $\overline{A}_{80:\overline{3}|}$.

Solution: (ix)

$$\overline{A}^{1}_{80:\overline{3}|} = \frac{i}{\delta} A^{1}_{80:\overline{3}|} = \frac{0.065}{\ln(1.065)}(0.5002507452) = 0.5163382634.$$

Example 3

Consider the life table

x	80	81	82	83	84	85	86
ℓ_x	250	217	161	107	62	28	0

Suppose that $i = 6.5\%$. *Calculate:*
(i) A_{80}. *(ii)* $A_{80:\overset{1}{\overline{3}|}}$. *(iii)* ${}_{3|}A_{80}$. *(iv)* $A^{1}_{80:\overline{3}|}$. *(v)* $A_{80:\overline{3}|}$. *(vi)* $\overline{A}_{80}$. *(vii)* $\overline{A}_{80:\overset{1}{\overline{3}|}}$. *(viii)* ${}_{3|}\overline{A}_{80}$. *(ix)* $\overline{A}^{1}_{80:\overline{3}|}$. *(x)* $\overline{A}_{80:\overline{3}|}$.

Solution: (x)

$$\overline{A}_{80:\overline{3}|} = \overline{A}^{1}_{80:\overline{3}|} + \overline{A}_{80:\overset{1}{\overline{3}|}} = 0.5163382634 + 0.35431941129 = 0.8706576747.$$

Theorem 3

Assuming a uniform distribution of deaths, we have that:
(i) $(I\overline{A})_x = \frac{i}{\delta}(IA)_x$.
(ii) $(\overline{IA})_x = \frac{i}{\delta}(IA)_x + \frac{i}{\delta}\left(\frac{1}{\delta} - \frac{1}{d}\right)A_x$.

To calculate actuarial present values for life insurance paid at the end of the m–thly interval of interval, we also need to interpolate life tables.

Theorem 4

Assuming a uniform distribution of deaths, we have that:

(i) $A_x^{(m)} = \frac{i}{i^{(m)}} A_x$.

(ii) $A^{(m)\,1}_{x:\overline{n}|} = \frac{i}{i^{(m)}} A^{1}_{x:\overline{n}|}$.

(iii) ${}_{n}|A_x^{(m)} = \frac{i}{i^{(m)}} \cdot {}_{n}|A_x$.

(iv) $A^{(m)}_{x:\overline{n}|} = \frac{i}{i^{(m)}} A^{1}_{x:\overline{n}|} + A_{x:\overset{1}{\overline{n}|}}$.

The factor $\frac{i}{i^{(m)}}$ has a life table interpretation. Suppose that $\ell_{x+k}-\ell_{x+k+1}$ deaths happens during the k–th year and $(\ell_{x+k}-\ell_{x+k+1})$ is multiple of m. Under the discrete annual life insurance the total benefit payment made at time k is $\ell_{x+k}-\ell_{x+k+1}$. Under a uniform distribution of deaths, $\frac{\ell_{x+k}-\ell_{x+k+1}}{m}$ deaths happen in each of the intervals $\left[k-1+\frac{j-1}{m}, k-1+\frac{j}{m}\right]$, $j=1,\dots,m$. The cashflow of payments under the m–th ly plan is

Payments	$\frac{\ell_{x+k}-\ell_{x+k+1}}{m}$	$\frac{\ell_{x+k}-\ell_{x+k+1}}{m}$	$\frac{\ell_{x+k}-\ell_{x+k+1}}{m}$	$\cdots$	$\frac{\ell_{x+k}-\ell_{x+k+1}}{m}$
Time	$k-1+\frac{1}{m}$	$k-1+\frac{2}{m}$	$k-1+\frac{3}{m}$	$\cdots$	$k-1+\frac{m}{m}$

The present value at time k of this cashflow is

$$(\ell_{x+k}-\ell_{x+k+1})s_{\overline{1}|i}^{(m)}=(\ell_{x+k}-\ell_{x+k+1})\frac{(1+i)^1-1}{\frac{i^{(m)}}{m}}=(\ell_{x+k}-\ell_{x+k+1})\frac{i}{i^{(m)}}$$

We get that the present value under the m–th ly plan is $s_{\overline{1}|i}^{(m)}=\frac{i}{i^{(m)}}$ times the present value under the annual plan.

Example 4

Consider the life table

x	80	81	82	83	84	85	86
ℓ_x	250	217	161	107	62	28	0

Suppose that $i = 6.5\%$. Calculate:

(i) $A_{80}^{(12)}$. (ii) ${}_{3|}A_{80}^{(12)}$. (iii) $\overset{(12)}{A}{}^{1}_{x:\overline{n}|}$. (iv) $A_{80:\overline{3}|}^{(12)}$.

Example 4

Consider the life table

x	80	81	82	83	84	85	86
ℓ_x	250	217	161	107	62	28	0

Suppose that $i = 6.5\%$. Calculate:

(i) $A_{80}^{(12)}$. (ii) ${}_{3|}A_{80}^{(12)}$. (iii) $\overset{(12)}{A}{}^{1}_{x:\overline{n}|}$. (iv) $A_{80:\overline{3}|}^{(12)}$.

Solution: (i)

$$A_{80}^{(12)} = \frac{i}{i^{(12)}} A_{80} = \frac{0.065}{12((1.065)^{1/12} - 1)}(0.8161901166)$$

$$=0.8402293189.$$

Example 4

Consider the life table

x	80	81	82	83	84	85	86
ℓ_x	250	217	161	107	62	28	0

Suppose that $i = 6.5\%$. Calculate:

(i) $A_{80}^{(12)}$. (ii) ${}_{3|}A_{80}^{(12)}$. (iii) $A_{x:\overline{n}|}^{1\,(12)}$. (iv) $A_{80:\overline{3}|}^{(12)}$.

Solution: (ii)

$$ {}_{3|}A_{80}^{(12)} = \frac{i}{i^{(12)}} \cdot {}_{3|}A_{80} = \frac{0.065}{12((1.065)^{1/12} - 1)}(0.3159393714) $$

$=0.3252447162.$

Example 4

Consider the life table

x	80	81	82	83	84	85	86
ℓ_x	250	217	161	107	62	28	0

Suppose that $i = 6.5\%$. Calculate:

(i) $A_{80}^{(12)}$. (ii) ${}_{3|}A_{80}^{(12)}$. (iii) $\overset{(12)}{A}{}^{1}_{x:\overline{n}|}$. (iv) $A_{80:\overline{3}|}^{(12)}$.

Solution: (iii)

$$\overset{(12)}{A}{}^{1}_{x:\overline{n}|} = A_{80}^{(12)} - {}_{3|}A_{80}^{(12)} = 0.8402293189 - 0.3252447162$$

$$= 0.5149846027.$$

Example 4

Consider the life table

x	80	81	82	83	84	85	86
ℓ_x	250	217	161	107	62	28	0

Suppose that $i = 6.5\%$. Calculate:

(i) $A_{80}^{(12)}$. (ii) ${}_{3|}A_{80}^{(12)}$. (iii) $\overset{(12)}{A}{}^{1}_{x:\overline{n}|}$. (iv) $A_{80:\overline{3}|}^{(12)}$.

Solution: (iv)

$$A_{80:\overline{3}|}^{(12)} = \overset{(12)}{A}{}^{1}_{x:\overline{n}|} + A_{80:\overset{1}{\overline{3}|}} = 0.5149846027 + 0.35431941129$$
$$= 0.869304014.$$

