Manual for SOA Exam MLC.

Chapter 5. Life annuities.

Section 5.4. n–year certain annuities.

Extract from:
"Arcones' Manual for the SOA Exam MLC. Fall 2009 Edition".
available at http://www.actexmadriver.com/

n–year certain life annuity–due

Definition 1

*A n–**year certain due life annuity** pays at the beginning of the year while either the individual is alive or the number of payments does not exceed n.*

Under the n–year certain due life annuity, the first n payments are guaranteed.

Definition 2

The present value of a n–year certain due life annuity for (x) with unit payment is denoted by $\ddot{Y}_{\overline{x:\overline{n}|}}$.

Definition 3

The actuarial present value of a n–year certain due life annuity for (x) with unit payment is denoted by $\ddot{a}_{\overline{x:\overline{n}|}}$.

We have that $\ddot{a}_{\overline{x:\overline{n}|}} = E[\ddot{Y}_{\overline{x:\overline{n}|}}]$.

Theorem 1

We have that

$$\ddot{Y}_{\overline{x:\overline{n}|}} = \ddot{a}_{\overline{\max(n,K_x)|}} = \ddot{a}_{\overline{n}|} + {}_{n}|\ddot{Y}_x = \ddot{a}_{\overline{n}|} + \sum_{k=n}^{\infty} Z_{x:\overline{k}|}^{\ \ 1},$$

$$\ddot{a}_{\overline{x:\overline{n}|}} = \ddot{a}_{\overline{n}|} \cdot {}_{n}q_x + \sum_{k=n+1}^{\infty} \ddot{a}_{\overline{k}|} \cdot {}_{k-1}|q_x = \ddot{a}_{\overline{n}|} + {}_{n}|\ddot{a}_x = \ddot{a}_{\overline{n}|} + \sum_{k=n}^{\infty} v^k \cdot {}_{k}p_x,$$

$$\mathrm{Var}(\ddot{Y}_{\overline{x:\overline{n}|}}) = \mathrm{Var}({}_{n}|\ddot{Y}_x).$$

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$$\mathrm{Var}(\ddot{Y}_{\overline{x:\overline{n}|}}) = \mathrm{Var}({}_{n}|\ddot{Y}_x).$$

Proof: Since payments go until $\max(n, K_x)$, $\ddot{Y}_{\overline{x:\overline{n}|}} = \ddot{a}_{\overline{\max(n,K_x)}|}$. If $K_x \le n$, the obtained cashflow consists of n payments. If $K_x > n$, the obtained cashflow consists of n payments as well as n deferred payments until the end of the year of death. Hence, $\ddot{Y}_{\overline{x:\overline{n}|}} = \ddot{a}_{\overline{n}|} + {}_{n}|\ddot{Y}_x$.

Example 1

A special pension plan pays \$30000 at the beginning of the year guaranteed for 10 years and continuing thereafter per life. Suppose that $i = 0.06$ and mortality follows the life table in the manual. Calculate the APV of this life insurance for (65).

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A special pension plan pays \$30000 at the beginning of the year guaranteed for 10 years and continuing thereafter per life. Suppose that $i = 0.06$ and mortality follows the life table in the manual. Calculate the APV of this life insurance for (65).

Solution: $\ddot{a}_{\overline{10}|0.06} = 7.801692274$. From the table, $A_{75} = 0.52383$. Hence, $\ddot{a}_{75} = \frac{1-0.52383}{6/106} = 8.412336667$. From the table, $\ell_{65} = 83114$ and $\ell_{75} = 66605$. So, $A_{65:\overline{10}|}^{\;\;\,1} = \frac{66605}{83114}(1.06)^{-10} = 0.4474803778$. Hence, ${}_{10|}\ddot{a}_{65} = (0.4474803778)(8.412336667) = 3.76435559$. The APV of this insurance is

$$(30000)(\ddot{a}_{\overline{10}|0.06} + {}_{10|}\ddot{a}_{65}) = (30000)(7.801692274 + 3.76435559)$$
$$= 346981.4359.$$

n–year certain life annuity–immediate

Definition 4

*A n–**year certain life annuity–immediate** pays at the end of the year while either the individual is alive or the number of payments does not exceed n.*

Under the n–year certain immediate life annuity, the first n payments are guaranteed.

Definition 5

The present value of a n–year certain immediate life annuity for (x) with unit payment is denoted by $Y_{\overline{x:\overline{n}|}}$.

Definition 6

The actuarial present value of a n–year certain immediate life annuity for (x) with unit payment is denoted by $a_{\overline{x:\overline{n}|}}$.

We have that $a_{\overline{x:\overline{n}|}} = E[Y_{\overline{x:\overline{n}|}}]$.

Theorem 2

We have that

$$Y_{\overline{x:\overline{n}|}} = a_{\overline{\max(n,K_{x-1})}|} = a_{\overline{n}|} + {}_n|Y_x = a_{\overline{n}|} + \sum_{k=n+1}^{\infty} Z_{x:\overset{1}{\overline{k}|}} = \ddot{Y}_{\overline{x:\overline{n+1}|}} - 1,$$

$$a_{\overline{x:\overline{n}|}} = a_{\overline{n}|} \cdot {}_nq_x + \sum_{k=n+1}^{\infty} a_{\overline{k-1}|} \cdot {}_{k-1}|q_x = a_{\overline{n}|} + {}_n|a_x$$

$$= a_{\overline{n}|} + \sum_{k=n+1}^{\infty} v^k \cdot {}_kp_x = \ddot{a}_{\overline{x:\overline{n+1}|}} - 1,$$

$$\mathrm{Var}(Y_{\overline{x:\overline{n}|}}) = \mathrm{Var}({}_n|Y_x).$$

n–year certain life continuous annuity

Definition 7

*A n–**year certain continuous life annuity** makes continuous payments while either an individual is alive or the length of the period of payments does not exceed n.*

Definition 8

The present value of a n–year certain continuous life annuity for (x) with unit rate is denoted by $\overline{Y}_{x:\overline{n}|}$.

Definition 9

The actuarial present value of a n–year certain continuous life annuity for (x) with unit rate is denoted by $\overline{a}_{\overline{x:\overline{n}|}}$.

We have that $\overline{a}_{\overline{x:\overline{n}|}} = E[\overline{Y}_{\overline{x:\overline{n}|}}]$.

Theorem 3

We have that

$$\overline{Y}_{\overline{x:\overline{n}|}} = \overline{a}_{\overline{\max(n,T_x)}|} = \overline{a}_{\overline{n}|} + {}_{n}|\overline{Y}_x,$$

$$\overline{a}_{\overline{x:\overline{n}|}} = \overline{a}_{\overline{n}|} \cdot {}_{n}q_x + \int_n^\infty \overline{a}_{\overline{t}|} \cdot {}_{t}p_x \mu_{x+t}\, dt = \overline{a}_{\overline{n}|} + {}_{n}|\overline{a}_x$$

$$= \overline{a}_{\overline{n}|} + \int_n^\infty v^t \cdot {}_{t}p_x\, dt,$$

$$\mathrm{Var}(\overline{Y}_{\overline{x:\overline{n}|}}) = \mathrm{Var}({}_{n}|\overline{Y}_x).$$

