Manual for SOA Exam MLC.

Chapter 6. Benefit premiums.

Section 6.9. Computing benefits premiums from a life table.

Extract from:
"Arcones' Manual for SOA Exam MLC. Fall 2010 Edition",
available at http://www.actexmadriver.com/

Interpolation for life insurance APV's.

Theorem 1

Assuming a uniform distribution of deaths, we have that:

(i) $A_x^{(m)} = \frac{i}{i^{(m)}} A_x$.

(ii) $A^{1\,(m)}_{x:\overline{n}|} = \frac{i}{i^{(m)}} A^1_{x:\overline{n}|}$.

(iii) ${}_n|A_x^{(m)} = \frac{i}{i^{(m)}} \cdot {}_n|A_x$.

(iv) $A^{(m)}_{x:\overline{n}|} = \frac{i}{i^{(m)}} A^1_{x:\overline{n}|} + A_{x:\overline{n}|}^{\ \ \,1}$.

Theorem 2

Assuming a uniform distribution of deaths, we have that:

(i) $\overline{A}_x = \frac{i}{\delta} A_x$.

(ii) $\overline{A}^1_{x:\overline{n}|} = \frac{i}{\delta} A^1_{x:\overline{n}|}$.

(iii) ${}_n|\overline{A}_x = \frac{i}{\delta} \cdot {}_n|A_x$.

(iv) $\overline{A}_{x:\overline{n}|} = \frac{i}{\delta} A^1_{x:\overline{n}|} + A_{x:\overline{n}|}^{\ \ \,1}$.

Interpolation of APV's of life annuities

Recall that:

$$\ddot{a}_x = \frac{1 - A_x}{d}, \quad \ddot{a}_x^{(m)} = \frac{1 - A_x^{(m)}}{d^{(m)}}, \quad \overline{a}_x = \frac{1 - \overline{A}_x}{\delta},$$

$$\ddot{a}_{x:\overline{n}|} = \frac{1 - A_{x:\overline{n}|}}{d}, \quad \ddot{a}_{x:\overline{n}|}^{(m)} = \frac{1 - A_{x:\overline{n}|}^{(m)}}{d^{(m)}}, \quad \overline{a}_{x:\overline{n}|} = \frac{1 - \overline{A}_{x:\overline{n}|}}{\delta}.$$

To find the interpolated values of life annuities, find the interpolated value of the corresponding life insurance, I am use the corresponding formula from above.

Interpolation of APV's of annual benefit premiums

The annual benefit premium P is $\frac{A}{\ddot{a}}$, where A and $\ddot{a}$ can have many different forms. Use the corresponding formula from above to find A and $\ddot{a}$.

Example 1

Consider the life table

x	80	81	82	83	84	85	86
ℓ_x	250	217	161	107	62	28	0

Assume that $i = 6.5\%$ *and uniform distribution of deaths over each year of death.*
Find $P_{80}^{(12)}$, *using that* $A_{80} = 0.8161901166$.

Solution: We have that

$$A_{80}^{(12)} = \frac{i}{i^{(12)}} A_{80} = \frac{0.065}{0.06314033132}(0.8161901166) = 0.8402293189,$$

$$\ddot{a}_{80}^{(12)} = \frac{1 - A_{80}^{(12)}}{d^{(12)}} = \frac{1 - 0.8402293189}{0.06103286385} = 2.543720348,$$

$$P_{80}^{(12)} = \frac{A_{80}}{\ddot{a}_{80}^{(12)}} = \frac{0.8161901166}{2.543720348} = 0.3208647198.$$

Example 2

Consider the life table

x	80	81	82	83	84	85	86
ℓ_x	250	217	161	107	62	28	0

Assume that $i = 6.5\%$ *and uniform distribution of deaths over each year of death. Find* $P(\overline{A}_{80})$ *using that* $A_{80} = 0.8161901166$.

Solution: We have that

$$\overline{A}_{80} = \frac{i}{\delta} A_{80} = \frac{0.065}{\ln(1.065)}(0.8161901166) = 0.8424379003,$$

$$\ddot{a}_{80} = \frac{1 - A_{80}}{d} = \frac{1 - 0.8161901166}{6.5/106.5} = 3.011654243,$$

$$P(\overline{A}_{80}) = \frac{\overline{A}_{80}}{\ddot{a}_{80}} = \frac{0.8424379003}{3.011654243} = 0.2797259686.$$

Example 3

Consider the life table

x	80	81	82	83	84	85	86
ℓ_x	250	217	161	107	62	28	0

Assume that $i = 6.5\%$ and uniform distribution of deaths over each year of death.
Find $\overline{P}(\overline{A}_{80})$, using that $A_{80} = 0.8161901166$.

Solution: We have that

$$\overline{A}_{80} = \frac{i}{\delta} A_{80} = \frac{0.065}{\ln(1.065)}(0.8161901166) = 0.8424379003,$$

$$\overline{P}(\overline{A}_{80}) = \frac{\delta \overline{A}_{80}}{1 - \overline{A}_{80}} = \frac{\ln(1.065)(0.8424379003)}{(1 - 0.8424379003)} = 0.3367076072.$$

