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Different dependence measures in spatial point processes*

Guillermo Ayala† and Amelia Simó‡

Abstract

A multivariate spatial point pattern consists of the locations of two or more types in a region: different tree or animal species, cases and controls of a disease, centroids of degenerated and non–degenerated nerve fibres in a cross–section of a nerve. The quantification of dependence amongst the different types of points is a basic question.

The measure of the dependence has been basically based on the product density or its integrated version, the cross $\mathbb{K}$ function, $\mathbb{K}_{12}$ ([3]). New measures of local dependence are proposed and studied in this paper. Their estimators and their expressions are considered under two different null hypotheses: independence and random labeling (i.e., one type of points is a random selection of the whole set of points while the other type of points are the remaining). They are compared with the product intensity and its integrated version, the cross $\mathbb{K}$ function.

A randomization test for testing the random labeling hypothesis is proposed and applied to two examples. These examples are the locations of maples and oaks in Lansing Woods [7] and the locations of normal and degenerated fibres in a vertical cross-section of a nerve from a rat [20].

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†*Mailing Address:* Departamento de Estadística e Investigación Operativa. Avda Vicent Andrés Estellés 1. 46100–Burjasot, Spain. *E–mail:* `Guillermo.Ayala@uv.es`

‡*Mailing Address:* Departament de Matemàtiques, Universitat Jaume I. Campus Riu Sec s/n. 12071–Castelló, Spain. *E–mail:* `Simo@mat.uji.es`.

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