

Linear Algebra- Practice Second Exam - ANSWERS

1.

a) Not valid: different number of columns

b) Not valid: different number of columns

c) $\begin{pmatrix} 5 & 5 \\ -1 & 0 \end{pmatrix}$

d) $\begin{pmatrix} 5 & 5 \\ -1 & 0 \end{pmatrix}$

e) $\begin{pmatrix} -6 & 3 \\ -5 & 3 \\ -11 & 5 \end{pmatrix}$

f) Not valid: the number of columns of B is not equal to the number of rows of A

g) $\begin{pmatrix} -6 & 9 \\ 10 & 5 \end{pmatrix}$

h) $\begin{pmatrix} 12 & 4 \\ -9 & -13 \end{pmatrix}$

2. a) No. Dimension of \mathcal{P}_2 is 3.

b) No. The polynomials are linearly dependent: the sum of the first two equals the third one.

c) Yes. Can be proven using row reduction, identifying \mathcal{P}_2 with \mathbb{R}^3 . (On the actual test you would need to provide the details).

3.

a) $\left\{ \begin{pmatrix} 1 \\ 2 \\ -1 \\ 3 \end{pmatrix}, \begin{pmatrix} 3 \\ 2 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \right\}$

b) $\{(1 \ 0 \ -2 \ 0 \ 1), (0 \ 1 \ 2 \ 0 \ 0), (0 \ 0 \ 0 \ 1 \ -3)\}$

c) $\left\{ \begin{pmatrix} 2 \\ -2 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ 0 \\ 3 \\ 1 \end{pmatrix} \right\}$

4.

a) $m \times n$

b) For every matrix A with n columns, $rk(A) + nullity(A) = n$. (Your instructor may ask for a different, equivalent formulation).

c) m

d) n

5. (Ask your instructor for what is considered sufficient justification)

a) No, this is not a subspace.

b) Yes, this is a subspace.

6. (Ask your instructor for what is considered sufficient justification)

a) False

b) True

c) False

d) True

7.

a) $-\vec{s}_1 + \vec{s}_2 - 3\vec{s}_3 = \vec{0}$

b) The "standard" choice is $\{\vec{s}_1, \vec{s}_2, \vec{s}_4\}$. Its span is the same as the span of all four given vectors (the whole \mathbb{R}^3). The answer is not unique. In fact, the way the problem is worded, any subset of the given four vectors that does not contain \vec{s}_1 , \vec{s}_2 , or \vec{s}_3 qualifies as a correct answer.