MATH 304 Midterm Examination I, Sample 1

There are **ten (10)** problems on **two** pages in this examination. All work must be shown. NO CALCULATORS allowed.

NOTE: Some of the vectors in this sample are listed horizontally to save space. You must use the notations appropriate for solving each problem.

Problem 1. Is the vector $\bar{v} = [1, -2, 0]$ in the span of the vectors $\bar{v}_1 = [1, 2, 3]$, $\bar{v}_2 = [2, 1, 6]$ and $\bar{v}_3 = [4, 5, 12]$? **Justify!**

Problem 2. Solve the system of linear equations and write your solution in a vector form.

$$\begin{cases} x + y + z & = & 7 \\ 2x + 4y - z & = & 9 \\ 2y - 3z & = & -5 \\ 5y - x - 10z & = & -22 \end{cases}$$

Problem 3.

a) Find an equation of the plane passing through the point $P = (0, 1, 2)$, perpendicular to the vector $\bar{v} = [2, 1, -1]$.

b) Find the parametric equation of the line of intersection of this plane and the plane $x + y - z = 1$.

Problem 4. List all possible reduced row echelon forms of (3×2) matrices.

Problem 5. Determine if the matrices $A = \begin{pmatrix} 2 & 1 \\ 3 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} -1 & 0 \\ 5 & 3 \end{pmatrix}$ are row-equivalent (can be obtained from each other by a sequence of elementary row transformations).

Problem 6. A function $f : \mathbb{R}^2 \to \mathbb{R}^3$ is given by the formula

$$f(x_1, x_2) = (2x_1 + 4x_2, 0, 4x_1 + 8x_2)$$

a) Is f one-to-one? Justify your answer.

b) Is f onto? Justify your answer.

Problem 7. A 4×3 matrix A has columns $\vec{v}_1$, $\vec{v}_2$, and $\vec{v}_3$. We are given that $\vec{v}_1$ and $\vec{v}_2$ are not multiples of each other, and that $\vec{v}_3 = 2\vec{v}_1 + 5\vec{v}_2$. Determine all possible reduced row echelon forms of A. Justify your answer.

Problem 8. Find the smallest possible $|\vec{u}|$, given that $\vec{u} \cdot \vec{v} = -2$, where $\vec{v} = [1, 0, 2, 2]$. Justify your answer.

Problem 9. Vectors $\vec{u}$ and $\vec{v}$ are solutions of some system of linear equations. Show that the vector $2\vec{u} - \vec{v}$ is also a solution of that system. **Hint:** You may want to use the relation between the solution set of the given system and the solution set of the corresponding homogeneous system.

Problem 10. Find all values of a, such that the system with the following augmented matrix $(A|b)$ is consistent:

$$(A|b) = \left(\begin{array}{cccc|c} 2 & 1 & 3 & 0 & 2 \\ 0 & 0 & 1 & 4 & a \\ 0 & 0 & 2 & 8 & 10 \end{array}\right)$$