

MATH 304 Midterm 1, Sample 1 - ANSWERS

Problem 1. No

Problem 2.

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 19/2 \\ -5/2 \\ 0 \end{bmatrix} + t \begin{bmatrix} -5/2 \\ 3/2 \\ 1 \end{bmatrix}$$

Problem 3.

a) $2x + y - z + 1 = 0$.

b) NOTE: there are multiple forms of the correct answer. One of the possible forms:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \\ 0 \end{bmatrix} + t \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

Problem 4.

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & * \\ 0 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}, \text{ where } * \text{ is any number.}$$

Problem 5. A and B are row-equivalent, since their Reduced Row

Echelon forms are both equal to $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

Problem 6.

a) No. For example, $f(0, 0) = f(2, -1)$

b) No. For example, $(0, 1, 0)$ is not equal to $f(x_1, x_2)$ for any (x_1, x_2) .

Problem 7.

$$\begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 5 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Possible justification. First and second columns of reduced row echelon form of A cannot be expressed as linear combination of previous columns. So

they are leading columns. The third column of the reduced row echelon form of A is a linear combination of the first and second column with coefficients 2 and 5.

Problem 8. Answer: $\frac{2}{3}$.

Possible justification. By the Cauchy-Schwarz Inequality,

$$3 = |\vec{u} \cdot \vec{v}| \leq |\vec{u}| \cdot |\vec{v}| = |\vec{u}| \cdot \sqrt{1 + 0 + 4 + 4}$$

Solving for $|\vec{u}|$, we get $|\vec{u}| \geq \frac{3}{2}$. The equality is obtained when $\vec{u} = -\frac{2}{9}\vec{v}$.

Problem 9.

Possible justification. First, note that $\vec{w} = \vec{u} - \vec{v}$ is a solution of the corresponding homogeneous system. Then note that $2\vec{u} - \vec{v}$ equals $\vec{u} + \vec{w}$, so it is a solution of the original system.

Problem 10. $a = 5$.