**MATH 304 Midterm Examination 1, Sample 2**

There are **eight (8)** problems on **two** pages in this examination. All work must be shown. NO CALCULATORS allowed.

NOTE: Some vectors in this sample are listed horizontally to save space. You must use the notations appropriate for solving each problem.

**Problem 1.** Use **Gauss-Jordan elimination** (reduced row echelon form) to solve the system of linear equations

$$\begin{cases} x & +y & +2z & -w & = & 2 \\ x & -y & & +3w & = & -4 \\ x & +2y & +3z & -3w & = & 5 \\ 4x & +3y & +7z & -2w & = & 5 \end{cases}$$

or explain why the system is inconsistent. If the system is consistent, write down the solution in a vector form. NO CREDIT will be given, if **any other method** is used.

**Problem 2.** Find all values of $k$ such that the given matrix is in its reduced row-echelon form:

$$\begin{pmatrix} k & 1 & 0 & 3 & 0 \\ 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

**Problem 3.** Determine if the vector $\begin{bmatrix} -2 \\ 5 \\ 17 \end{bmatrix}$ is a linear combination of the vectors

$$\mathbf{v_1} = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}, \mathbf{v_2} = \begin{bmatrix} 11 \\ 6 \\ 7 \end{bmatrix}, \mathbf{v_3} = \begin{bmatrix} -1 \\ 2 \\ 7 \end{bmatrix}, \text{ and } \mathbf{v_4} = \begin{bmatrix} 2 \\ 0 \\ -2 \end{bmatrix}.$$

Explain in details your conclusion.

**Problem 4.** Given three points $A = (1, 2, 3),\ B = (-1, 5, 0)$

and $C = (2, -3, 2)$.

a) Write parametric equations of the plane containing points $A$, $B$ and $C$.

b) Determine if the triangle with the vertices $A$, $B$ and $C$ is acute, obtuse or right triangle. Give detailed explanation.

**Problem 5.** Find all values of $k$ such that the matrices

$$A = \begin{pmatrix} 1 & k+1 \\ -1 & 3k-2 \end{pmatrix} \text{ and } B = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

are **not row-equivalent**. Give detailed explanation.

**Problem 6.** For what values of $k$, will the following system have (a) no solutions, (b) a unique solution, (c) infinitely many solutions?

$$\begin{cases} x - 2y + 3z = 2 \\ x + y + z = k \\ 2x - y + 4z = k^2 \end{cases}$$

**Problem 7.** Find all vectors in $\mathbb{R}^4$ that are simultaneously orthogonal to the following vectors:

$$[1, 1, 0, -1], \quad [4, 2, 2, 1]$$

**Problem 8.** Given a matrix $A$ in a reduced row echelon form (RRE), which of the following statements are **always** true? Explain or provide a counterexample for each one.

a) The matrix obtained from $A$ by removing the last column is in RRE.

b) The matrix obtained from $A$ by removing the first column is in RRE.

c) The matrix obtained from $A$ by removing the last row is in RRE.

d) The matrix obtained from $A$ by removing the first row is in RRE.