# MATH 304 Midterm Examination 1, Sample 3

There are **nine (9)** problems on **two** pages in this examination. All work must be shown. NO CALCULATORS allowed.

NOTE: Some of the vectors in this sample are listed horizontally to save space. You must use the notations appropriate for solving each problem.

**Problem 1.** Find the reduced row echelon form of a matrix

$$\begin{bmatrix} 0 & 1 & 2 & 2 \\ 1 & 2 & 1 & 2 \\ 1 & 3 & 3 & 1 \end{bmatrix}$$

**Problem 2.** Find all values of $k$ such that the angle between the vectors $\vec{u} = [1, 1]$ and $\vec{v} = [1, k]$ is $\frac{\pi}{3}$.

**Problem 3.** Find all values of $k$ such that the vector $[1, 2, k, 1]$ lies in he span of the vectors $[1, 0, 1, 2]$, $[2, 1, 1, 0]$, and $[2, 1, 0, 1]$.

**Problem 4.** a) Find all values of the parameter $a$ such that the system of linear equations in $x$, $y$, and $z$ with the following augmented matrix is consistent.

$$\left( \begin{array}{ccc|c} 2 & 1 & 3 & 2 \\ 0 & a & 1 & 3 \\ 0 & 0 & 0 & (a^2 - a) \end{array} \right)$$

b) For each of the values of $a$ from part (a) solve the system.

**Problem 5.** If $[1, 3, 5]$ and $[2,6,10]$ are solutions of some system of linear equations, does this system of equations have to be homogeneous? Justify your answer.

**Problem 6.** Can a linear system with 3 equations and 4 variables have (a) no solutions (b) unique solution (c) infinitely many solutions? Justify your answer.

**Problem 7.** For what values of the parameter $k$ is the following matrix non-singular?

$$\left( \begin{array}{ccc} 1 & 2 & 2 \\ 3 & 2 & k \\ 1 & 3 & 1 \end{array} \right)$$

**Problem 8.** Suppose a vector $\vec{w}$ is a linear combination of vectors $\vec{u}$ and $\vec{v}$ with coefficients 2 and 3. Does this imply that the vector $5\vec{u} + 4\vec{w}$ is a linear combination of vectors $\vec{v}$ and $\vec{w}$? Justify your answer.

**Problem 9.** Find all vectors in the span of the vectors $[1, 2, 0, 1]$ and $[2, 0, 2, 1]$ that are perpendicular to the vector $[-1, 1, 1, 2]$.