MATH 304 Midterm 1 Sample 3-ANSWERS

Problem 1.

$$\begin{bmatrix} 1 & 0 & -3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Problem 2. $k = \sqrt{3} - 2$

Problem 3. $k = 10$

Problem 4. a) $a = 0$ or $a = 1$
b) For $a = 0$:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -\frac{7}{2} - \frac{1}{2}t \\ t \\ 3 \end{bmatrix}$$

For $a = 1$:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} - t \\ 3 - t \\ t \end{bmatrix}$$

Problem 5. Yes.

One possible justification is that the difference of two vectors, $[-1, -3, -5]$, has to be a solution of the corresponding homogeneous system. Therefore, $[0, 0, 0]$ which is its sum with $[1, 3, 5]$ has to be a solution of the original system. Thus the original system is homogeneous.

Alternatively, one can look at any equation of our system, $a_1x_1 + a_2x + a_3x_3 = b$. We are given that $a_1 + 3a_2 + 5a_3 = b$ and $2a_1 + 6a_2 + 10a_3 = b$. Subtracting from the second equation twice the first, we get that $b = 0$.

Problem 6.

a) Yes. For example, one of the equations can be contradictory: $0 = 1$. Many other, less obvious, examples are possible.

b) No. Because the number of free variables is at most 3, at least one of the 4 variables will be free. So if a system is consistent, it will have infinitely many solutions.

c) Yes. In particular, any homogeneous system will have infinitely many solutions from the argument in part (b).

Problem 7. All real k except 10.

Problem 8. Yes.
We are given that $\vec{w} = 2\vec{u} + 3\vec{v}$. Solving for $\vec{u}$, we get $\vec{u} = -\frac{3}{2}\vec{v} + \frac{1}{2}\vec{w}$.
So $5\vec{u} + 4\vec{w} = -\frac{15}{2}\vec{v} + \frac{13}{2}\vec{w}$.

Problem 9. $\begin{bmatrix} 4t \\ -4t \\ 6t \\ t \end{bmatrix}$