MATH 304 Midterm Examination 2, Sample 1

There are **seven (7)** problems on **two** pages in this examination. All work must be shown. NO CALCULATORS allowed.

NOTE: Some of the vectors in this sample are listed horizontally to save space. You must use the notations appropriate for solving each problem.

Problem 1.(10 pts) Show that the following matrices form a basis of the space of all upper-triangular 2×2 matrices:

$$\left\{ \begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix}, \begin{bmatrix} 0 & 3 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 2 & 3 \\ 0 & 2 \end{bmatrix} \right\}$$

Problem 2.(30 pts) Matrix B is given as follows.

$$B = \begin{bmatrix} 2 & 1 & 0 & 4 & 0 \\ 4 & 2 & 0 & 7 & -2 \\ 4 & 2 & 9 & 11 & 3 \\ 2 & 1 & 3 & 5 & 1 \end{bmatrix}$$

a) Find a basis for the column space of B.
b) Find a basis for the row space of B.
c) Find a basis for the nullspace of B.
d) Find the rank and the nullity of B.

Problem 3.(15 pts) Is the following set of polynomials a subspace of the space of polynomials of degree at most 2?
a) The set of all polynomials $f(x)$ such that $f(1) = 0$
b) The set of all polynomials $f(x)$ such that $f(2) = 1$

Problem 4.(10 pts) Suppose that columns of a matrix A are linearly dependent. Does this imply that the rows of A are also linearly dependent? Explain or provide a counterexample.

Problem 5.(10 pts) Suppose that A is a square matrix and its columns are linearly dependent. Does this imply that the rows of A are also linearly dependent? Explain or provide a counterexample.

Problem 6.(10 pts) Write down a matrix of a linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^2$ that rotates the plane counterclockwise by the angle $\frac{\pi}{2}$ and then reflects around the line $y = x$. (We represent $\mathbb{R}^2$ as $\{\begin{bmatrix} x \\ y \end{bmatrix}\}$).

Problem 7.(15 pts) Is it true that for every polynomial $f(x)$ of degree 2 the polynomials $f(x)$, $f'(x)$, and $f''(x)$ form a basis of P_2? Justify or provide a counterexample.