MATH 304 Midterm 2, Sample 2 - ANSWERS

Problem 1. a) Ask your instructor

b) $\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$ (Any two linearly independent vectors would work: the Range is the entire $\mathbb{R}^2$).

c) $\{x^2 - 2x + 1\}$ (If the variable is x).

Problem 2. a) Identifying $M_{2,2}$ with $\mathbb{R}^4$, we can see that V is a null space of some matrix. (One can also prove that V is a subspace directly from the definition, using properties of matrix multiplication).

b) $\left\{ \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right\}$

Problem 3. (Ask your instructor for what is considered a sufficient justification)
a) False
b) True
c) True
d) May be true or false, depending on the vectors
e) False
f) True

Problem 4. a) $\begin{bmatrix} 5 & 2+2k \\ 7 & 2+3k \\ 10 & 4+4k \end{bmatrix}$

b) $k = 4$

c) $\left\{ \begin{bmatrix} 5 \\ 7 \\ 10 \end{bmatrix} \right\}$ (or any non-zero multiple of it)

d) $\{[5\ 10]\}$ (or any non-zero multiple of it, for example $\{[1\ 2]\}$)

e) $\left\{ \begin{bmatrix} -2 \\ 1 \end{bmatrix} \right\}$ (or any non-zero multiple of it)

Problem 5. S is not a subspace. (There are many counter-examples that can be used to show that it is not closed under scalar multiplication or addition).

Problem 6.

a) The standard example is $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$.

b) One possible example is $\begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}$.

Problem 7. Suppose $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. Then

$$A^t A = \begin{bmatrix} a^2 + c^2 & ab + cd \\ ab + cd & b^2 + d^2 \end{bmatrix}$$

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Since this is the zero matrix, we must have $a^2 + c^2 = 0$ and $b^2 + d^2 = 0$. For real a, b, c, d this implies that $a = b = c = d = 0$.