MATH 304 Midterm Examination 3, Sample 2

There are **seven (7)** problems on **two** pages in this examination. All work must be shown. NO CALCULATORS allowed.

Problem 1. Find the standard matrix of $T : \mathbb{R}^2 \to \mathbb{R}^2$ if its matrix in the basis $\left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$ is $\begin{pmatrix} 3 & 1 \\ 0 & 3 \end{pmatrix}$.

Problem 2. Find the determinant of the matrix $F = \frac{1}{2}(C^T)$ if

$$C = \begin{pmatrix} 1 & 1 & 1 & 0 \\ 0 & 2 & 2 & 1 \\ 1 & 0 & 3 & 0 \\ 0 & 1 & 0 & 4 \end{pmatrix}$$

Problem 3. a) Show that the transformation $T : \mathbb{R}^4 \to \mathbb{R}^4$, given by

$$T\left(\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}\right) = \begin{bmatrix} x + 2y - z + w \\ 2y - w \\ x + z - y \\ 3x + z \end{bmatrix}$$

is linear.

b) Determine if T is invertible. If it is invertible, find the matrix of the inverse transformation.

Problem 4. Given the matrix

$$A = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 0 & 0 \\ 1 & 2 & 5 \end{pmatrix}$$

represent A and A^{-1} as products of elementary matrices.

Problem 5. Suppose A is an $n \times n$ real matrix. For each of the following statements determine if it is always true. Justify.

a) If A is diagonalizable, then A is diagonal.

b) If $\det(A) = 0$ then $\lambda = 0$ is an eigenvalue for A.

c) If A is invertible, then the columns of A form a basis of $\mathbb{R}^n$.

d) If A is a product of elementary matrices then it is similar to the identity matrix.

e) If A is similar to the identity matrix, then A is the identity matrix.

Problem 6. If A is the standard matrix of the linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^2$ given by the reflection about the line $y = -3x$, find two linearly independent eigenvectors of A and the corresponding eigenvalues GEOMETRICALLY. Any other method will receive NO CREDIT.

Problem 7. The matrix A is given by

$$A = \begin{pmatrix} 0 & 2 & -2 \\ -2 & 5 & -1 \\ -2 & 1 & 3 \end{pmatrix}$$

a) Find the characteristic polynomial of the matrix A.

b) Find all eigenvalues of A and the corresponding eigenvectors and eigenspaces.

c) Find an invertible matrix P and a diagonal matrix D so that $A = PDP^{-1}$ or explain why this is impossible.