MATH 304 FINAL EXAMINATION - Sample 1

Problem 1. a) Use **Gauss-Jordan elimination** (reduced row echelon form) to solve the system of linear equations

$$\begin{cases} \begin{array}{rrrrrr} x & & +2z & -2w & = & -3 \\ -x & +y & & +w & = & 4 \\ 2x & +y & +z & & = & -5 \\ -x & +y & +5z & -4w & = & 4 \end{array} \end{cases}$$

or explain why the system is inconsistent. If the system is consistent, write down the solution in a vector form. NO CREDIT will be given, if **any other method** is used.

b) Determine if the vectors $\begin{bmatrix} 1 \\ -1 \\ 2 \\ -1 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 2 \\ 0 \\ 1 \\ 5 \end{bmatrix}$ and $\begin{bmatrix} -2 \\ 1 \\ 0 \\ -4 \end{bmatrix}$

are linearly independent or not. Explain.

c) Determine if the vector $\begin{bmatrix} -3 \\ 4 \\ -5 \\ 4 \end{bmatrix}$ is in the span of vectors $\begin{bmatrix} 1 \\ -1 \\ 2 \\ -1 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 2 \\ 0 \\ 1 \\ 5 \end{bmatrix}$ and $\begin{bmatrix} -2 \\ 1 \\ 0 \\ -4 \end{bmatrix}$. Explain.

Problem 2. a) For the matrix $A = \begin{pmatrix} 2 & 1 & 1 & 3 & -1 \\ 1 & 0 & -1 & -1 & 2 \\ 4 & 1 & -1 & 1 & 3 \end{pmatrix}$

find each of the following:

i) basis of $col(A)$; ii) basis of $row(A)$;

iii) dimension of $col(A)$; iv) dimension of $row(A)$; v) $nullity(A)$.

b) Find a basis for the orthogonal complement $W^{\perp}$ of $W = span\,\{[2,1,1,3,-1],\ [1,0,-1,-1,2],\ [4,1,-1,1,3]\}$

Problem 3. The transformation $T : P_2 \to P_2$ is given by the formula $T(p)(x) = p(x) + 2p'(x)$, where $p(x)$ is any polynomial in P_2.

a) Show that T is linear by checking the defining conditions for a transformation to be linear.

b) Find the matrix of T in the basis $\{1, x, x^2\}$.

c) Determine if the transformation T is invertible or not.
If it is invertible, find the matrix of the inverse transformation T^{-1} in the basis $\{1, x, x^2\}$. If it is not invertible, explain why.

Problem 4. Find the inverse of the matrix

$$A = \begin{pmatrix} 2 & 0 & 1 & -1 \\ 0 & 1 & 1 & 1 \\ -1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{pmatrix}$$

or explain why it does not exist.

Problem 5. The matrix A is given by

$$A = \begin{pmatrix} 1 & 2 & 3 \\ -1 & -2 & -3 \\ 1 & 2 & 3 \end{pmatrix}$$

Determine if A is diagonalizable or not. If it is, find an invertible matrix P and a diagonal matrix D such that $D = P^{-1}AP$.
(You DO NOT have to find P^{-1}).
If it is not diagonalizable, explain why.

Problem 6. Suppose W is a subspace of $\mathbb{R}^5$, such that
$W = span\,\{[1, 0, -1, 1, 1],\ [1, 2, 1, 0, 4],\ [-1, 2, 1, 0, 10]\}$.

a) Use the Gram-Schmidt process to construct an orthogonal basis of W.

b) Use the basis of W found in part a) to find the orthogonal projection $proj_W(\mathbf{v})$ of vector $\mathbf{v} = [0, 18, 0, 36, 0]$ onto W.

Problem 7. Determine if each of the statements below is TRUE or FALSE. Circle your choice and give the explanation for your answer.

a) Any two eigenvectors corresponding to the same eigenvalue must be linearly dependent

TRUE FALSE

Explanation:

b) Any two eigenvectors corresponding to different eigenvalues must be linearly independent

TRUE FALSE

Explanation:

c) For any square invertible $n \times n$ matrix A

$$\det((2A)^{-1}) = \frac{1}{2 \cdot \det(A)}$$

TRUE FALSE

Explanation:

d) If columns of a square matrix add up to 0, then the matrix is not invertible

TRUE FALSE

Explanation:

e) The inverse of an elementary matrix is an elementary matrix

TRUE FALSE

Explanation:

f) For any square $n \times n$ matrix A, if $\det A = 0$ then A is not diagonalizable

TRUE FALSE

Explanation: