

## MATH 304 Final Examination, Sample 1-ANSWERS

### Problem 1.

$$\text{a) } \begin{bmatrix} x \\ y \\ z \\ t \end{bmatrix} = \begin{bmatrix} -3 \\ 1 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \\ 1 \\ 1 \end{bmatrix} t$$

b) The given vectors are the columns of the matrix  $A$  in part a).

$rk(A) = 3 < 4$  (number of vectors), so the vectors are linearly dependent.

c) The system  $(A|\vec{b})$  is consistent, so  $rk(A) = rk(A|\vec{b})$ , So the given vector is in the span of the other vectors.

### Problem 2.

a)

$$\text{(i) Basis of } \text{col}(A): \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

(ii) Basis of  $\text{row}(A)$ :  $[1, 0, -1, -1, 2]$ ,  $[0, 1, 3, 5, -5]$

(iii)  $\dim(\text{col}(A)) = 2$

(iv)  $\dim(\text{row}(A)) = 2$

(v)  $\text{nullity}(A) = 3$

$$\text{b) Basis of } W^\perp \text{ is } \begin{bmatrix} 1 \\ -3 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -5 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 5 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

### Problem 3.

a) Ask your instructor.

$$\text{b) } \{1, x, x^2\}[T]_{\{1, x, x^2\}} = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{pmatrix}.$$

$$\text{c) } \{1, x, x^2\}[T^{-1}]_{\{1, x, x^2\}} = \begin{pmatrix} 1 & -2 & 8 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{pmatrix}.$$

**Problem 4.**

$$A^{-1} = \begin{pmatrix} 0 & 1 & -1 & 0 \\ -1 & 5 & -4 & -2 \\ 1 & -3 & 3 & 1 \\ 0 & -1 & 1 & 1 \end{pmatrix}$$

**Problem 5.**

$$D = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2 \end{pmatrix}, \quad P = \begin{pmatrix} 2 & -3 & 1 \\ -1 & 0 & -1 \\ 0 & 1 & 1 \end{pmatrix}$$

(the matrices  $D$  and  $P$  are not unique)

**Problem 6.** (The answers are listed horizontally to save space):

a) Orthogonal basis of  $W$  :  $[1, 0, -1, 1, 1]$ ,  $[0, 2, 2, -1, 3]$ ,  $[-3, -2, -1, 0, 2]$

b)  $[15, 4, -7, 9, 5]$

**Problem 7.** a) F; b) T; c) F; d) T; e) T; f) F