

MATH 304 Final Examination, Sample 2-ANSWERS

Problem 1.

$$\frac{1}{\sqrt{2}}$$

Problem 2.

The vectors are linearly dependent. $2\vec{v}_1 - \vec{v}_2 + \vec{v}_3 = \vec{0}$

Problem 3.

- (i) $k = 1$
- (ii) $k \neq -2, 1$
- (iii) $k = -2$

Problem 4.

- (i) $(1, -2, 0, 5, 0), (0, 0, 1, 3, 0), (0, 0, 0, 0, 1)$ (the basis is not unique)

- (ii) $\begin{bmatrix} 2 \\ 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \\ 2 \\ 4 \end{bmatrix}$ (the basis is not unique)

- (iii) $\begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -5 \\ 0 \\ -3 \\ 1 \\ 0 \end{bmatrix}$ (the basis is not unique)

- (iv) $\text{rk}(A) = 3$
- (v) $\text{nullity}(A) = 2$

Problem 5.

$$\begin{pmatrix} 1 & 0 & 1 \\ 1 & 2 & 0 \\ 2 & 0 & 2 \\ 2 & 5 & 3 \end{pmatrix}$$

Problem 6.

$$x = 2, y = 1$$

Problem 7.

$$D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{pmatrix}, \quad P = \begin{pmatrix} 0 & -1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$

(the matrices D and P are not unique)

Problem 8.

a) Orthogonal basis: $\begin{bmatrix} 2 \\ -1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -3 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 7 \\ 3 \\ 1 \end{bmatrix}$

(the basis is not unique)

b) $\frac{1}{84} \begin{bmatrix} 77 \\ 335 \\ 3 \\ 173 \end{bmatrix}$

Problem 9. a) F; b) F; c) F; d) F; e) F; f) F; g) F