MATH 304 Final Examination, Sample 3-ANSWERS

Problem 1.

a) $$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 5 \\ -7 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ -5 \\ 0 \\ 1 \end{bmatrix} t + \begin{bmatrix} -2 \\ 6 \\ 1 \\ 0 \end{bmatrix} s$$

b) i) Basis of $col(A)$ is $\begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$;

ii) Basis of $row(A)$ is $[1, 0, 2, -1]$, $[0, 1, -6, 5]$;
iii) $rk(A) = 2,\ nullity(A) = 2$;
iv) Yes, the system is consistent.

Problem 2.

a) $$A^{-1} = \begin{pmatrix} 1/2 & -1/2 & 2 \\ -1/2 & -1/2 & 3 \\ 0 & 1 & -4 \end{pmatrix}$$

b) Yes, the vectors are the columns of the invertible matrix.

Problem 3.

a) $$A_T = \begin{pmatrix} 4 & \frac{\sqrt{3}}{2} \\ 4\sqrt{3} & -\frac{1}{2} \end{pmatrix}.$$

Problem 4. a) $\lambda_1 = \lambda_2 = 0,\ \lambda_3 = 4$.

b) For $\lambda = 0$, basis of E_0 is $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -2 \\ 1 \end{bmatrix} \right\}$;

For $\lambda = 4$, basis of E_4 is $\left\{ \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} \right\}$

c) For $\lambda = 0$, both algebraic and geometric multiplicities are equal to 2. For $\lambda = 4$, both algebraic and geometric multiplicities are equal to 1.

d) A is diagonalizable, $D = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 4 \end{pmatrix}$, $P = \begin{pmatrix} 1 & 0 & 3 \\ 0 & -2 & 2 \\ 0 & 1 & 1 \end{pmatrix}$

(the matrices D and P are not unique)

Problem 5. a) Basis of $W^{\perp}$ is $\left\{ \begin{bmatrix} -1 \\ 2 \\ -1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -3 \\ 2 \\ 1 \\ 0 \end{bmatrix} \right\}$.

b) Orthogonal basis of W is $\left\{ \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 2 \\ 2 \\ -2 \end{bmatrix} \right\}$.

Problem 6. a) F; b) T; c) T; d) F; e) F; f) F; g) F