MATH 304 Final Examination, Sample 4

Problem 1. Show that A is diagonalizable. Then find A^{2012}.

$$A = \begin{pmatrix} 4 & 2 & 2 \\ 2 & 1 & 1 \\ -8 & -4 & -4 \end{pmatrix}$$

Problem 2. Suppose V is the space of all linear combinations with real coefficients of functions $f_1(x) = e^x$, $f_2(x) = xe^x$, and $f_3(x) = x^2e^x$.

a) Show that $T(f)(x) = f'(x) - f(x)$ defines a linear transformation from V to V.

b) Find the matrix of T in the basis $\{f_1, f_2, f_3\}$.

c) Find the kernel and the range of T.

Problem 3. a) Determine whether the given matrix A is invertible. If it is invertible, find its inverse:

$$A = \begin{pmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 2 & 3 & 3 \end{pmatrix}$$

b) Represent the matrix A as a product of elementary matrices or show that it is not possible.

Problem 4. Find $det(\frac{1}{3}A)$, if $A = \begin{pmatrix} 1 & 2 & -1 & 0 & 1 \\ 0 & 0 & 2 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 2 & 1 \\ 0 & -3 & 0 & 4 & 10 \end{pmatrix}$

Problem 5. Find bases for $row(A)$, $col(A)$ and $null(A)$, and the rank and nullity of A, if

$$A = \begin{pmatrix} -1 & 3 & 4 & 0 & -2 \\ 0 & 1 & -3 & -1 & 2 \\ -3 & 7 & 18 & 2 & -10 \end{pmatrix}$$

Problem 6. For ach of the following subsets of $\mathbb{R}^2$ determine if they are subspaces. Justify your answer.

a) $S = \{(x, y)|x = y^2\}$
b) $S = \{(x, y)|x = y + 1\}$
c) $S = \{(x, y)|x = 2y\}$
d) $S = \{(x, y)|x^2 - 2y^2 = 0\}$
e) $S = \{(x, y)|x^2 + 2y^2 = 0\}$

Problem 7. Find an orthogonal basis of $\mathbb{R}^4$, containing vector $[1, 1, 1, 1]$.

Problem 8. Determine if each of the statements below is TRUE or FALSE. Circle your choice and give the explanation for your answer.

a) Every orthogonally diagonalizable matrix is invertible

TRUE FALSE

Explanation:

b) Every orthogonal matrix is invertible

TRUE FALSE

Explanation:

c) For any square $n \times n$ matrices A and B, if A and B have the same Reduced Row-Echelon form then $\det A = \det B$

TRUE FALSE

Explanation:

d) Two eigenvectors corresponding to the same eigenvalue must be linearly independent

TRUE FALSE

Explanation:

e) Two eigenvectors corresponding to different eigenvalues must be linearly dependent

TRUE FALSE

Explanation:

f) For any four vectors $\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}, \mathbf{v_4}$ in $\mathbb{R}^3$, the vector $\mathbf{v_4}$ must be a linear combination of the vectors $\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}$

TRUE FALSE

Explanation: