

MATH 304 Final Examination, Sample 5-ANSWERS

**Problem 1.** a) 
$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -1 \\ -1 \\ 1 \\ 0 \end{bmatrix} t + \begin{bmatrix} -1 \\ 2 \\ 0 \\ 1 \end{bmatrix} s$$

b) The vectors are linearly dependent.

**Problem 2.** a) 
$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$$

b) 
$$A^{-1} = \begin{pmatrix} 1 & 3/2 & -1/2 \\ 3 & 7/2 & -3/2 \\ -6 & -17/2 & 7/2 \end{pmatrix}$$

**Problem 3.**

a) 
$$A_T = \begin{pmatrix} -\frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix};$$
 b)  $T$  is invertible,  $A_{T^{-1}} = \begin{pmatrix} -\frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix}$

**Problem 4.** a)  $\lambda_1 = 0, \lambda_2 = \lambda_3 = 2.$

b) For  $\lambda = 0$ , basis of  $E_0$  is  $\left\{ \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\};$

For  $\lambda = 2$ , basis of  $E_2$  is  $\left\{ \begin{bmatrix} 1/2 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -3/2 \\ 1 \\ 0 \end{bmatrix} \right\}$

(or  $\left\{ \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} -3 \\ 2 \\ 0 \end{bmatrix} \right\}$ )

c)  $A$  is diagonalizable,  $D = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$ ,  $P = \begin{pmatrix} 0 & 1 & -3 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{pmatrix}$

(the matrices  $D$  and  $P$  are not unique)

d)  $A$  is not orthogonally diagonalizable (since  $A$  is not symmetric)

**Problem 5.** a)  $\dim(\text{row}(A)) = \dim(\text{col}(A)) = \text{rk}(A) = 2$ ,  
 $\text{nullity}(A) = \dim(\text{null}(A)) = 3$ .

Basis of  $\text{row}(A)$  :  $\{[1, 0, -1, 1, 3], [0, 1, 1/6, -1/3, 1/6]\}$  (or  $\{[1, 0, -1, 1, 3], [2, 6, -1, 0, 7]\}$ );

Basis of  $\text{col}(A)$  :  $\left\{ \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 6 \\ 6 \end{bmatrix} \right\}$ .

Basis of  $\text{null}(A)$  :  $\left\{ \begin{bmatrix} 1 \\ -1/6 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 1/3 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ -1/6 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$ .

b) Basis of  $W^\perp$  is the basis of  $\text{null}(A)$  :  $\left\{ \begin{bmatrix} 1 \\ -1/6 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 1/3 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ -1/6 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$ .

c)  $v_1 = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$ ,  $v_2 = \begin{bmatrix} -1 \\ 4 \\ 7 \end{bmatrix}$

d)  $\begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix}$

**Problem 6.** a) F; b) T; c) F; d) T; e) T; f) F; g) F