Seventh Upstate NY Number Theory Conference
Talk Abstracts

Plenary Talks
Science Library, Room 212

1) Speaker: Alexandra Shlapentokh

Title: On First-Order Definability and Decidability Problems Over Number Fields and their Infinite Algebraic Extensions

Abstract: We will discuss some recent developments connected to several problems of definability and decidability over number fields and infinite algebraic extensions of $\mathbb{Q}$. In particular, we will address the results concerning the big subrings of number fields and so-called p-bounded extensions of $\mathbb{Q}$.

2) Speaker: Joseph H. Silverman (Brown University)

Title: Complexity Growth in Algebraic Geometry, Number Theory, and Dynamics

Abstract: A rough measure of the complexity of a mathematical object is the number of bits it takes to describe some aspect of the object. For example, the degree of a rational map $f : \mathbb{P}^N - - > \mathbb{P}^N$ is a measure of f's geometric complexity, while the height of a rational point P in $\mathbb{P}^N(\mathbb{Q})$ is a measure of P's arithmetic complexity. We consider the n'th iterate f^n of f and investigate the growth rate of the geometric complexity of f^n and the arithmetic complexity of $f^n(P)$ as n increases. This leads to the theory of dynamical and arithmetic degrees, concerning which I will describe some recent theorems of interest and several intriguing open questions.

3) Speaker: Melvyn Nathanson

Title: Problems in additive number theory and some applications of commutative algebra

Abstract: This will be a survey of open problems in additive number theory, with examples of the application of classical theorems in commutative algebra to obtain additive results.

4) Speaker: Michael Filaseta

Title: 4959866698915122609810424451291 8

Abstract: If p is a prime written in base 10 as $d_r d_{r-1} \dots d_0$, then the polynomial $f(x) = d_r x^r + \cdots + d_1 x + d_0$ is irreducible. This result due to Arthur Cohn has been generalized in a number of directions. We will survey some recent work on these generalizations done with my former students Morgan Cole, Scott Dunn and Sam Gross. We will also provide a simple proof of Cohn's theorem where the digits d_j are allowed to be replaced by non-negative integers $\leq 10^{15}$.

5) Speaker: Tom Hales

Title: The Spherical Hecke algebra, partition functions, and motivic integration

Abstract: This talk will describe a proof of the LanglandsShelstad fundamental lemma for the spherical Hecke algebra for every unramified p-adic reductive group G in large positive characteristic. The fundamental lemma is a collection of identities of integrals that were proved by Ngo Bao Chau and that are needed to put the Arthur-Selberg trace formula in a form that is suited for applications to number theory. Our work extends these identities from the unit element of the Hecke algebra to the full spherical Hecke algebra. The proof is based on the transfer principle for constructible motivic integration, which allows identities that are proved for one field to be moved to another field. To carry this out, we introduce a general family of partition functions attached to the complex L-group of the unramified p-adic group G. Our partition functions specialize to Kostants q-partition function for complex connected groups and also specialize to the Langlands local L-function of a spherical representation. This is joint work with Bill Casselman and Jorge Cely.

6) Speaker: Jeffrey Lagarias

TItle: Polynomial splitting measures, braid group cohomology and hyperplane arrangements

Abstract: This talk starts from the number theoretic problem of determining splitting probabilities for different factorizations of monic degree n polynomials over p-adic fields into irreducibles, conditioned on having square-free factorization type. In work with Benjamin L. Weiss (U. Maine) we showed these probabilities for a fixed splitting type, viewed as a function of p, interpolate as rational functions (of a parameter z) which are Laurent polynomials. By identifying splitting types with conjugacy classes of the symmetric group S_n these probabilities define class functions on S_n. The speaker subsequently studied the case $z = 1$, corresponding to "field with one element", and found the resulting measure values have multiplicative arithmetical structure, and also (after rescaling) are for each n the character of a virtual representations of S_n having small support. Recent work with Trevor Hyde (U. Michigan) determined that the (rescaled) individual Laurent coefficients are characters of (virtual) representations of the symmetric group S_n, arising from part of the cohomology of the pure braid group, viewed as an S_n-module, acting on the complement of the A_n-hyperplane arrangement. We discuss arithmetic consequences of this identification.

Contributed Talks
Session 1 (Saturday Morning)
Science Library, Room 310

1) Speaker: Wade Hindes

Title: Integral points in dynamical orbits

Abstract: Over a number field K, a celebrated result of Silverman states that if $\phi(x) \in K(x)$ is a rational function whose second iterate is not a polynomial, then the set of integral points in the orbit $\mathrm{Orb}_\phi(P) = \{\phi^n(P)\}_{n\geq 0}$ is finite for all basepoints $P \in \mathbb{P}^1(K)$. In this talk, I will discuss Silverman's finiteness theorem from several new perspectives, including an "average" version and a generalization to global fields of finite characteristic.

2) Speaker: George Grell

Title: Galois Groups of Iterated Quadratic Rational Maps whose Critical Orbits Collide

Abstract: In 2013, Pink studied properties of an inverse limit of Galois groups of iterated quadratic rational maps with infinite postcritical orbits whose critical points eventually meet. We use the theory of ramification groups to give an explicit representation of the Galois groups after an arbitrary number of iterations. In certain cases we are able to use this to give an expression for the fixed point proportion of the standard group action on a binary rooted tree. We are also able to extend the main result to cubic polynomials with similar properties.

3) Speaker: Jamie Juul

Title: The arboreal Galois representation of a PCF cubic polynomial

Abstract: Let K be a field, $f(x)$ a polynomial of degree $d > 1$ with coefficients in K, and $a \in K$. The roots of the polynomials $f^n(x) - a$, which are the pre-images of a under $f^n(x)$, have a natural structure as a d-ary rooted tree. We can study the action of the absolute Galois group of K on this tree structure. For a generic polynomial, this will give the full automorphism group of the tree. However in certain cases, such as when the polynomial is post-critically finite, this representation is known to have infinite index. In this talk, we will give a complete description of this group for the polynomial $f(x) = -2x^3 + 3x^2$ and base point $a = 3$ over the rational numbers.

4) Speaker: Wayne Peng

Title: How to Image a One Dimensional Perfectoid Space, and its Application on Dynamical System.

Abstract: The characteristic 0 spaces and the characteristic p spaces are very different by the algebraic structure, but they are still connected

in some way. Like, some conjectures over number fields have analogue statements over the field of characteristic p where are theorems. The model theory could explain this phenomena by taking p to infinity but not a fixed p. Until 2012, Peter Scholze gave a paper, Perfecotid Spaces, established the relation between the two vary different worlds by a homeomorphism map for a fixed prime p. In this talk, we will construct a one-dimensional perfectoid space from an adic space which as a set is a part of Berkovich space with infinitely many type-5 points. Then, we will show you some recent developments, by Junyi Xie and mine, on the dynamical systems using the perfectoid spaces.

Science Library, Room 306

5) Speaker: Brian Hwang

Title: Arithmetic moduli spaces of abelian varieties and limit linear series on curves

Abstract: The Abel-Jacobi map associates an algebraic curve of genus g with its Jacobian: a g-dimensional abelian variety (with a principal polarization). This induces a map between the respective moduli spaces, but as the genus gets larger, the codimension of the image of this map gets larger as well, so it may seem hopeless to study moduli spaces of abelian varieties this way. However, what if we introduce some extra structure and try to vary the input curve in some way? Well show that certain constructions from the theory of limit linear series on curves—which studies how certain line bundles on smooth curves behave as the curves degenerate to particular kinds of reducible curves—can be used to extract arithmetic information about moduli spaces of abelian varieties. In particular, if we rigidify the target moduli space with certain types of level structure to obtain varieties like the modular curve $X_0(p)$, we can use objects from the theory of limit linear series to study infinitesimal neighborhoods of a point in the image, even in some cases where the moduli space is not smooth. Surprisingly, this turns out to be enough information to, say, calculate the local zeta function of the entire moduli space of abelian varieties (with that kind of level structure).

6) Speaker: Fan Zhou

Title: Voronoi formula and balanced Voronoi formulas

Abstract: Voronoi formulas are Poisson-style summation formulas for automorphic forms (on GL(n)). Firstly, we present a proof of Voronoi formula for coefficients of a large class of L-functions, in the style of the classical converse theorem of Andre Weil. Our formula applies to full-level cusp forms, Rankin-Selberg convolutions, and some isobaric sums. Secondly, we present the balanced Voronoi formula, with

its both sides twisted by hyper-Kloosterman sums. They are joint work with Eren Mehmet Kiral and Stephen Miller, respectively.

7) Speaker: Jeffrey Hatley

Title: Rank Parity for Congruent Modular Forms

Abstract: Suppose f and g are two modular forms with isomorphic mod p Galois representations. Several recent results have used (algebraic) techniques from Iwasawa theory to describe how the (analytic) ranks of these two forms are related. This talk will give an overview of these results, with an emphasis on the special case of elliptic curves (for clarity and concreteness) in the supersingular setting.

8) Speaker: Jack Buttcane

Title: Higher weight on GL(3).

Abstract: Classical automorphic forms are generally attached to subgroups of $\Gamma = SL(2,\mathbb{Z})$ and are functions on $\Gamma\backslash\mathbb{H} = \Gamma\backslash PSL(2,\mathbb{R})/SO(2,\mathbb{R})$. The come in two flavors: holomorphic modular forms and (spherical or weight 0) Maass forms. These two types of automorphic forms are both special cases of Maass forms with weight, and they collectively generate a basis of $L^2(\Gamma\backslash PSL(2,\mathbb{R}))$. The spherical Maass forms have been generalized to subgroups of $SL(n,\mathbb{Z})$, and these are currently a popular topic of study, particularly on $SL(3,\mathbb{Z})$. This talk will describe the generalization to Maass forms with weight on $SL(3,\mathbb{Z})$, the new types of non-spherical forms that arise, and what is currently known about them.

Science Library, Room 302

9) Speaker: Alicia Marino

Title: Strictly k-Regular Quadratic Forms

Abstract: An integral quadratic form is said to be strictly k-regular if it primitively represents all quadratic forms of k variables that are primitively represented by its genus. We show that, for k¿1, there are finitely many inequivalent positive definite primitive integral quadratic forms of k+4 variables that are strictly k-regular. Our result extends a recent finiteness result of Andrew Earnest et al. (2014) on strictly regular quadratic forms of 4 variables.

10) Speaker: Travis Morrison

Title: Non-norms of global fields are diophantine

Abstract: For a global field K, a subset A of K^n is diophantine over K if it is defined by a first-order existential formula in the language of rings, or in other words, is the image of a variety under a K-morphism. I will discuss recent joint work with Kirsten Eisentraeger in which we prove that the set of pairs (x,y) such that x is not a norm of $K(\sqrt{y})$

is diophantine over K. Equivalently, the set of pairs (x, y) such that $xs^2 + yt^2 = 1$ has no K-rational points is diophantine over K.

11) Speaker: Sonny Arora

Title: Constructing Picard Curves with Complex Multiplication

Abstract: For applications to cryptography, one is interested in constructing curves of genus less than or equal to 3 over finite fields whose Jacobians have complex multiplication. Several algorithms exist in the cases of elliptic curves and genus 2 curves, however, less in known in the genus 3 case. An interesting test case for such an algorithm is the case of Picard curves which are genus 3 curves of the form $y^3 = f(x)$ where f is a separable polynomial of degree 4. We discuss some difficulties encountered in the genus 3 case and present an algorithm to construct Picard curves with complex multiplication using a Chinese Remainder Theorem approach.

12) Speaker: Anna Haensch

Title: Almost universal ternary sums of polygonal numbers

Abstract: In 1796 Gauss showed that every natural number can be written as the sum of three triangular numbers. In 2009, Chan and Oh determined when a weighted sum of triangular numbers (i.e. triangular numbers with coefficients) represents all but finitely many natural numbers. We say such a sum is almost universal. In this talk we will determine when a sum of three generalized m-gonal numbers is almost universal. We will approach this question first from an algebraic, and then from analytic point of view, exploiting the capabilities of each method, and realizing new connections between the machinery.

Session 2 (Saturday Afternoon)

Science Library, Room 310

13) Speaker: Claire Burrin

Title: Dedekind sums associated to arbitrary lattices in SL(2,R)

Abstract: Dedekind sums are surprisingly omnipresent; they pop up in geometry, physics, combinatorics and of course number theory. The study of generalized Dedekind sums, which are attached to cuspidal hyperbolic surfaces - or, equivalently, cofinite Fuchsian groups - offers some explanation. I will give an overview of these generalized Dedekind sums and of some of their aspects.

14) Speaker: Kwangho Choiy

Title: Multiplicities in restriction of tempered representations of p-adic groups

Abstract: We shall discuss the multiplicity of tempered representations of a p-adic group, when restricted to its closed subgroup sharing

the same derived group. It relates to the multiplicity occurring in corresponding component groups in Langlands dual sides, under the assumptions of the tempered local Langlands conjecture and internal structure of tempered L-packets. This provides an extensive formula of the multiplicities for tempered representations of a p-adic group and generalizes Hiraga and Saito's relevant work of inner forms of $SL(n)$.

15) Speaker: Edmund Karasiewicz

Title: Fourier Coefficients of a Minimal Parabolic Eisenstein Series on the Double Cover of SL(3) over Q

Abstract: Brubaker-Bump-Friedberg-Hoffstein studied the non-degenerate Fourier coefficients of minimal parabolic Eisenstein series on the n-fold cover of SL(3) over a number field K containing the 2n-th roots of unity. Using some results of Miller, we compute the n=2 case over Q and describe how the results compare with those of Brubaker-Bump-Friedberg-Hoffstein. Additionally, we will describe the degenerate terms and the behavior at the bad prime p=2.

16) Speaker: Keping Huang

Title: The Greatest Common Divisors for the Orbits under Polynomial Functions

Abstract: Assume Vojta's Conjecture. Suppose $a, b, \alpha, \beta \in \mathbb{Z}$, and $f(x), g(x) \in \mathbb{Z}[x]$ are polynomials of degree $d \geq 2$. Assume that the sequence $(f^{\circ n}(a), g^{\circ n}(b))_n$ is generic and α, β are not exceptional for f, g respectively, we prove that for each given $\varepsilon > 0$, there exists constant $C = C(\varepsilon, a, b, \alpha, \beta, f, g) > 0$, such that for all $n \geq 1$, we have

$$\gcd(f^{\circ n}(a) - \alpha, g^{\circ n}(b) - \beta) \leq C \cdot \exp(\varepsilon \cdot d^n).$$

17) Speaker: Joseph Hundley

Title: On holomorphy of adjoint L-functions

Abstract: The adjoint L-function of an irreducible cuspidal automorphic representation π of $GL_n(\mathbb{A})$ ($\mathbb{A}$ the adeles of a number field), may be defined as $L(s, \pi, Ad) = L(s, \pi \times \tilde{\pi})/\zeta(s)$, where $\tilde{\pi}$ is the contragredient. It is expected that this L-function is always entire. We discuss an approach to proving this in the special case $n = 3$, which is based on the integral representation for the partial adjoint L-function due to Ginzburg. Our approach also applies to quasisplit unitary groups and to twisted adjoint L-functions.

Science Library, Room 306

18) Speaker: Jaiung Jun

Title: Geometry of hyperfields.

Abstract: Hyperrings and hyperfields are algebraic structures which generalize commutative rings and fields. In this talk, we aim to introduce these exotic structures and illustrate how hyperrings and hyperfields show up and fit into the classical theory, in particular, algebraic geometry and combinatorics.

19) Speaker: Vaidehee Thatte

Title: Ramification Theory for Arbitrary Valuation Rings in Positive Characteristic

Abstract: Our goal is to develop ramification theory for arbitrary valuation fields, that is compatible with the classical theory of complete discrete valuation fields with perfect residue fields. We consider fields with more general (possibly non-discrete) valuations and arbitrary (possibly imperfect) residue fields. The defect case, i.e., the case where there is no extension of either the residue field or the value group, gives rise to many interesting complications.We present some new results for Artin-Schreier extensions of valuation fields in positive characteristic. These results relate the "higher ramification ideal" of the extension with the ideal generated by the inverses of Artin-Schreier generators via the norm map. We also introduce a generalization and further refinement of Kato's refined Swan conductor in this case. Similar results are true in the mixed characteristic case.

20) Speaker: Álvaro Lozano-Robledo

Title: A probabilistic model for the distribution of ranks of elliptic curves over Q

Abstract: In this talk, we propose a new probabilistic model for the distribution of ranks of elliptic curves in families of fixed Selmer rank, and compare the predictions with previous results, and with the databases of curves over the rationals that we have at our disposal. In addition, we document a phenomenon we refer to as Selmer bias that seems to play an important role in the data and in our models.

21) Speaker: Erik Wallace

Title: Rank gain of Jacobians over finite Galois Extensions

Abstract: The topic discussed in this talk was the result of a collaboration with Bo-Hae Im. Let K be a number field, and let $\mathcal{X} \to \mathbb{P}^1_K$ be a degree p-covering branched only at 0, 1, and ∞. If K is a field containing a primitive p-th root of unity then the covering of $\mathbb{P}^1$ is Galois over K, and if p is congruent to 1 mod 6, then there is an automorphism σ of $\mathcal{X}$ which cyclically permutes the branch points. Under these assumptions, we show that the Jacobian of both $\mathcal{X}$ and $\mathcal{X}/\sigma$ gain rank over infinitely many linearly disjoint cyclic degree p-extensions of K.

We also show the existence of an infinite family of elliptic curves whose j-invariants are parametrized by a modular function on $\Gamma_0(3)$ and that gain rank over infinitely many cyclic degree 3-extensions of $\mathbb{Q}$.

22) Speaker: Daniel Miller

Title: Galois representations with specified SatoTate distributions

Abstract Akiyama and Tanigawa have a conjecture describing the rate of convergence of the sets Satake parameters of an elliptic curve to the Sato–Tate measure. I construct Galois representations $\rho\colon G_{\mathbf{Q}} \to \mathrm{GL}_2(\mathbf{Z}_l)$, for which the "fake Satake parameters" converge arbitrarily slowly to any specified measure. These Galois representations are infinitely ramified, but the density of the set of ramified primes can be controlled.

Science Library, Room 302

23) Speaker: Alisa Sedunova

Title: Integral points on algebraic curves over function fields.

Abstract: Bombieri and Pila proved bounds for the number of integral points on an algebraic curve lying in a small box in terms of the degree of the curve and the size of a box. We extend their technique to the case of $F_q[T]$ and further show how to improve this result for the particular case of elliptic curves. We use ideas of Silverman, repulsion techniques and results about sphere packings.

24) Speaker: Krzysztof Pawelec

Title: Mean Values and Value Distribution of $L'/L(1+it,\pi)$

Abstract: For ρ, a cuspidal automorphic representation of $GL_m(\mathbb{A}_{\mathbb{Q}})$, there is an associated L-function, $L(\rho, s)$. We study value distribution of its logarithmic derivative on 1-line, $\frac{L}{L}(\rho, 1+it)$. We are able to prove that in some sense $\frac{L}{L}(\rho, 1+it)$ has almost normal distribution with mean 0 and variance $\frac{(\log\log(t))^2}{(\exp((\log\log(t))^2)}$. An essential ingredient of the proof is the fact that our function of interest can be approximated by Dirichlet polynomial with coefficients supported on prime powers.

25) Speaker: Cezar Lupu

Title: Analytic aspects in the evaluation of some multiple zeta values and multiple Hurwitz zeta values.

Abstract: In this talk, we shall discuss some new results in the evaluation of some multiple zeta values (MZV). More precisely, we provide evaluation of the Hoffman basis in terms of an infinite rational series involving even values of the Riemann zeta function. We also discuss similar results for the multiple t-values and alternating multiple zeta values. The multiple zeta values (Euler-Zagier sums) were introduced independently by Hoffman and Zagier in 1992 and they play a crucial

role at the interface of analysis, number theory, combinatorics, algebra and physics.

26) Speaker: Junxian Li

Title: A lower bound for the least prime in an arithmetic progression.

Abstract: Fix k a positive integer, and let ℓ be coprime to k. Let $p(k,\ell)$ denote the smallest prime equivalent to $\ell \pmod k$, and set $P(k)$ to be the maximum of all the $p(k,\ell)$. In this joint work with Kyle Pratt and George Shakan, we show that for almost every k one has $P(k) \gg \phi(k)\log k \log_2 k \log_4 k/\log_3 k$. This improves an earlier bound of Pomerance, and answers a question of Ford, Green, Konyagin, Maynard, and Tao. We also give a heuristic which suggests that

$$\liminf_k \frac{P(k)}{\phi(k)\log^2 k} = 1.$$

27) Speaker: Simon Rydin Myers

Title: Diophantine inequalities in many variables

Abstract: Consider the equation $f = 0$, where f is a polynomial of degree d, with integral coefficients, in n variables. If n is large and f is in a suitable sense nonsingular, then one can apply the circle method to estimate the number of solutions in integers of bounded size. If we allow f to have real coefficients, one might expect a parallel result counting solutions of the inequality $|f| < 1$. We state a theorem of this type, and outline the new ingredients required for the proof.