Upstate New York Online Number Theory Colloquium

Speaker: Michael Filaseta

Title: On a dense universal Hilbert set

Abstract: A *universal Hilbert set* is an infinite set $\mathcal{S} \subseteq \mathbb{Z}$ having the property that for every $F(x, y) \in \mathbb{Z}[x, y]$ which is irreducible in $\mathbb{Q}[x, y]$ and satisfies $\deg_x(F) \geq 1$, we have that for all but finitely many $y_0 \in \mathcal{S}$, the polynomial $F(x, y_0)$ is irreducible in $\mathbb{Q}[x]$. The existence of universal Hilbert sets is due to P. C. Gilmore and A. Robinson in 1955, and since then a number of explicit examples have been given. Universal Hilbert sets of density 1 in the integers have been shown to exist by Y. Bilu in 1996 and P. Dèbes and U. Zannier in 1998. In this talk, we discuss a connection between universal Hilbert sets and Siegel's Lemma on the finiteness of integral points on a curve of genus ≥ 1, and explain how a result of K. Ford (2008) implies the existence of a universal Hilbert set $\mathcal{S}$ satisfying

$$|\{m \in \mathbb{Z} : m \notin \mathcal{S}, |m| \leq X\}| \ll \frac{X}{(\log X)^{\delta}},$$

where $\delta = 1 - (1 + \log\log 2)/(\log 2) = 0.086071\ldots$. This is joint work with Robert Wilcox.